

A Supersymmetric Grand Unified Model with Noncompact Horizontal Symmetry

Naoki Yamatsu*

Department of Physics, Indiana University, Bloomington, IN 47405, USA

Abstract

In a supersymmetric $SU(5)$ grand unified model with a horizontal symmetry $SU(1,1)$, we discuss spontaneous generation of generations to produce three chiral generations of quarks and leptons and one generation of higgses by using one structure field with a half-integer spin of $SU(1,1)$ and two structure fields with integer spins. In particular, the colored higgses can disappear without fine-tuning. The difference of the Yukawa coupling matrices between the down-type quarks and charged leptons is discussed. We show that some special $SU(1,1)$ weight assignments include R -parity as a discrete subgroup, and R -parity remains even after we take into account the $SU(1,1)$ breaking effects from all the VEVs of the structure and matter fields. The assignments forbid the baryon and/or lepton number violating terms except a superpotential dimension-4 operator including a coupling of two lepton doublets and two up-type higgses. We discuss how to generate sizable neutrino masses. We show that the proton decay derived from the colored higgses is highly suppressed.

1 Introduction

The implications of the fundamental theory of nature from low energy phenomena come from the problems of 't Hooft's naturalness [1] and fine-tuning [2,3]. The problems are the window to hidden structures of nature. One of the naturalness problems of hierarchical mass structures of quarks and leptons suggests the existence of horizontal symmetry [4–7]. The strong CP problem [8–11] implies the spontaneous discrete symmetry breaking containing P/CP symmetry [12–17] or existence of the axion [18–22]. For a review, see, e.g., Ref. [23]. The fine-tuning problem of quadratic divergence of the higgs mass term suggests the existence of supersymmetry [24–26], extra-dimension [27–29], or technicolor [30,31].

The implications are not only the issues of naturalness, but also the quantization of charges, the anomaly cancellation of the standard model (SM) gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y (= G_{\text{SM}})$ by each generation of quarks and leptons at low energies [32], the unification of three gauge coupling constants at the unification scale, and the matter unification of quarks and leptons in SM for one or two representations in grand unified groups. They seem to suggest that one of the hidden structures of nature is some unified gauge symmetry [33,34]. As is well-known, candidates for the grand unified gauge symmetry are simple groups, such as $SU(5)$ [34–36], $SU(6)$ [37,38], $SO(10)$ [39–42], and E_6 [43–46]. For a review, see e.g., Refs. [47–49]. Any grand unified model explains the quantization of charges, and some of them explain the anomaly cancellation and the SM gauge coupling unification. Here we will focus on the $SU(5)$ unified group.

As is well-known, the non-supersymmetric $SU(5)$ grand unified model [34] that contains the minimal numbers of quarks, leptons, and higgs predicts rapid proton decay via X and Y gauge bosons. As long as the colored higgs mass is $O(M_{\text{GUT}})$, the Yukawa coupling constants of the first and second generations of quarks and leptons coupling to the colored higgs are smaller than the gauge coupling constant. The most strict restriction for the proton decay via the X and Y

*Electronic address: nyamatsu@indiana.edu

gauge bosons comes from the mode $p \rightarrow \pi^0 e^+$. By using the chiral Lagrangian technique, the lifetime is given by [50]

$$\tau(p \rightarrow \pi^0 e^+) \rightarrow 1.1 \times 10^{36} \times \left(\frac{M_V}{10^{16} \text{GeV}} \right)^4 \left(\frac{0.003 \text{GeV}^3}{\alpha} \right)^2 \text{ years}, \quad (1)$$

where M_V is the X and Y gauge boson mass and α is a hadron matrix element. When we use the gauge bosons masses $M_V \sim M_{\text{GUT}} \sim 10^{15} \text{ GeV}$ and a hadron matrix element $\alpha = 0.003 \text{ GeV}^3$, we obtain $\tau(p \rightarrow \pi^0 e^+) = 1.1 \times 10^{32} \text{ years}$. From the latest result from the super-Kamiokande [51], the lifetime $\tau(p \rightarrow \pi^0 e^+) > 8.2 \times 10^{33} \text{ years}$ at 90 %C.L. Thus, as is well-known, the non SUSY $SU(5)$ GUT model seems to be ruled out.

Fortunately, in the minimal $SU(5)$ SUSY GUT model [2, 52–54] the GUT scale M_{GUT} becomes $O(10^{16}) \text{ GeV}$. Substituting $M_V = 10^{16} \text{ GeV}$ in Eq. (1), we obtain the proton lifetime $\tau(p \rightarrow \pi^0 e^+) = 1.1 \times 10^{36} \text{ years}$. Thus, the lifetime satisfies the current bound. However, it is also known that the minimal $SU(5)$ SUSY GUT model suffers from rapid proton decay induced from the colored higgses [50, 55–57]. According to Ref. [56], the colored higgs masses must be greater than 10^{17} GeV for any $\tan \beta$ by using the recent super-Kamiokande result for the lifetime $\tau(p \rightarrow K^+ \bar{\nu}) > 3.3 \times 10^{33} \text{ years}$ at 90% C.L. [58] when we assume that the sfermion masses are less than 1 TeV. Thus, the colored higgses must have the effective mass greater than $O(10^{17}) \text{ GeV}$. On the other hand, the doublet higgs must have $O(m_{\text{SUSY}})$. This is known as a doublet-triplet splitting problem [59–66].

In addition, the minimal $SU(5)$ GUT model gives the unacceptable relation between the Yukawa coupling constants of down-type quarks and charged leptons without taking into account the higher dimensional operators including the nonvanishing VEVs of the adjoint representation. To break the minimal GUT relation of the Yukawa coupling constants between down-type quarks and charged leptons, roughly speaking, we can classify two methods; one is to consider the higher dimensional operators including the $SU(5)$ adjoint higgs field; another is to introduce the higher dimensional representations, such as Georgi-Jarlskog manner (see, e.g., Ref. [67] for an $SU(5)$ Non-SUSY GUT model and Ref. [68] for an $SU(5)$ SUSY GUT model); and another is to mix the above ways.

Even when we consider SUSY GUT models, they do not give us any insight about the hierarchy of the Yukawa couplings and the number of chiral generations of quarks, leptons and higgses. The mass parameters at the GUT scale in the minimal supersymmetric standard model (MSSM) [69–71] are given by Ref. [72] for several values of $\tan \beta$ by using the renormalization group equations of the two-loop gauge couplings and the two-loop Yukawa couplings assuming an effective SUSY scale of 500 GeV. For $\tan \beta = (10, 38, 50)$, the coupling constants of the third generation of the up-type quark, the down-type quark, and the charged lepton at the GUT scale are $y_t \simeq (0.48, 0.49, 0.51)$, $y_b \simeq (0.051, 0.23, 0.37)$, and $y_\tau \simeq (0.070, 0.32, 0.51)$, respectively. When we normalize the Yukawa coupling constant of the third generations equal to one, the mass parameters of first, second and third generations of the up-type quark, the down-type quark, and the charged lepton for $\tan \beta = 10$ are $(\tilde{y}_1, \tilde{y}_2, \tilde{y}_3) \simeq (6.7 \times 10^{-6}, 2.5 \times 10^{-3}, 1)$, $(1.0 \times 10^{-3}, 1, 2 \times 10^{-2}, 1)$, and $(2.5 \times 10^{-4}, 6 \times 10^{-2}, 1)$, respectively, where the subscript of \tilde{y}_a ($a = 1, 2, 3$) stands for the number of generation. The values are almost the same for $\tan \beta = 38$ and 50.

The hierarchical structures of the Yukawa couplings of quarks and leptons strongly suggest the existence of a hidden structure of nature. There have been many attempts to understand the origin of the hierarchical structures and/or generations by using horizontal symmetries G_H [4–7]: e.g., non-abelian group symmetries [73, 74], an abelian group $U(1)$ [5, 75], and a noncompact nonabelian group symmetry $SU(1, 1)$ [76], where the noncompact group $SU(1, 1)$ is a special pseudo-unitary group [77, 78].

In this article, we discuss an $\mathcal{N} = 1$ supersymmetric vectorlike $SU(5)$ GUT model with a noncompact horizontal symmetry $SU(1, 1)$ to solve the above problems. We summarize the main results of previous studies of $\mathcal{N} = 1$ supersymmetric vectorlike models with a horizontal

symmetry $SU(1, 1)$ [76, 79–83]. The number of chiral generations of matter fields, such as quarks, leptons and higgses are determined by the spontaneous symmetry breaking of the horizontal symmetry $SU(1, 1)$, called the spontaneous generation of generations [76]. Through the mechanism, the doublet-triplet splitting of higgses can be realized without fine-tuning and also unreasonably suppressed tiny mass parameters [79, 81]. When the horizontal symmetry is unbroken, the original Yukawa coupling matrices of matter fields are completely determined by $SU(1, 1)$ symmetry. The Yukawa coupling constants of the chiral matter fields at low energy are controlled by the $SU(1, 1)$ symmetry and the $SU(1, 1)$ breaking vacua. Each structure of Yukawa couplings of three chiral generations of quarks and leptons has hierarchical structure [76, 79, 81, 83]. The problematic superpotential dimension-3 operators $\hat{Q}\hat{L}\hat{D}^c$, $\hat{D}^c\hat{D}^c\hat{U}^c$, $\hat{L}\hat{L}\hat{E}^c$ are automatically forbidden, where in the MSSM these operators are forbidden by R -parity [84] (or matter parity [85]) to prevent rapid proton decay (For a review, see, e.g., Ref. [86]). The dangerous superpotential dimension-4 operators $\hat{Q}\hat{Q}\hat{Q}\hat{L}$ and $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$ are also not allowed where the usual R -parity cannot forbid these operators [79, 81].

Here we mention an $\mathcal{N} = 1$ supersymmetric noncompact gauge theory since our model is based on an $\mathcal{N} = 1$ supersymmetric noncompact gauge theory. As is well-known, renormalizable noncompact gauge theories have ghost problems; at least one gauge field has a negative metric in the canonical kinetic term, which indicates the wrong sign and this is the physical ghost; the structure fields belonging to the finite dimensional representations also have the physical ghosts. A solution of this problem is discussed in Ref. [87] by using an $\mathcal{N} = 1$ supersymmetric model with a noncompact gauge group $SU(1, 1)$ that has noncanonical Kähler function and gauge kinetic function with linear representation of $SU(1, 1)$ gauge transformation. At least at classical level, the Lagrangian has gauge and Kähler metrics positive definite at proper vacua, and thus no ghost fields exist at the vacua. For another solution of this problem, see, e.g., Refs. [88, 89].

The main purpose of this paper is to show that an $SU(5)$ SUSY GUT model with the noncompact horizontal symmetry $SU(1, 1)$ naturally satisfies current proton decay experiments, solves the doublet-triplet mass splitting problem, and avoids the unrealistic GUT relation for Yukawa couplings. In addition, we will see that this model can accommodate R -parity as a discrete subgroup of the horizontal symmetry and naturally produce the hierarchical Yukawa coupling structures of quarks and leptons.

This paper is organized as follows. In Sec. 2, we first set up our model. In Sec. 3, we discuss spontaneous generation of generations to produce three chiral generations of quarks and leptons and one generation of higgses by using one structure field with a half-integer spin of $SU(1, 1)$ and two structure fields with integer spins proposed in Ref. [83], where structure fields are chiral superfields with the finite dimensional representation of $SU(1, 1)$. In particular, we find that the colored higgses can disappear without fine-tuning. In Sec. 4, we see the structure of the Yukawa couplings, especially, how to realize the difference of the Yukawa coupling matrices between the down-type quarks and charged leptons. In Sec. 5, we discuss how to generate the effective μ -term of higgses. In Sec. 6, we discuss the baryon and/or lepton number violation. First, we identify which baryon and/or lepton number violating terms are allowed by using $SU(1, 1)$ symmetry and R -parity. We show that some specific $SU(1, 1)$ weight assignments include R -parity as a discrete subgroup, and R -parity remains even after we take in account of the $SU(1, 1)$ breaking effects from all the VEVs of the structure and matter fields. An article [80] suggested that a $G_{\text{SM}} \times SU(1, 1)$ model with particular matter content allows only Type-II seesaw mechanism [90–92] to generate neutrino masses. In general, not only Type-II seesaw mechanism but also Type-I and Type-III seesaw mechanisms are allowed, where the $SU(1, 1)$ weight assignments are severely constrained. We see that the proton decay derived from the colored higgses is highly suppressed. Section 7 is devoted to a summary and discussion.

Field	\hat{F}_{10}	\hat{F}'_{10}	\hat{G}_{5^*}	\hat{G}'_{5^*}	\hat{H}_{u5}	\hat{H}_{d5^*}	\hat{S}_1	\hat{R}_1	\hat{N}_1	\hat{T}_{15}	\hat{A}_{24}
$SU(5)$	10	10	5*	5*	5	5*	1	1	1	15	24
$SU(1,1)$	$+\alpha$	$+\alpha'$	$+\beta$	$+\beta'$	$-\gamma$	$-\delta$	$+\eta$	$+\lambda$	$+\xi$	$-\tau$	$+\zeta$
(R -parity)	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$-$

Table 1: The quantum numbers of matter fields in the $SU(5) \times SU(1,1)$ model are given in the table, and the model has also their conjugate fields. The Greek letters of the $SU(1,1)$ row represent the highest or lowest eigenvalues of $SU(1,1)$ weights. The negative value is the highest weight and the positive value is the lowest weight of $SU(1,1)$.

2 Setup of an $SU(5) \times SU(1,1)$ model

We construct an $SU(5)$ SUSY GUT model with horizontal symmetry $SU(1,1)$ that contain vectorlike matter contents. We introduce the matter fields

$$\hat{F}_{10}, \hat{F}'_{10}, \hat{G}_{5^*}, \hat{G}'_{5^*}, \hat{H}_{u5}, \hat{H}_{d5^*}, \hat{S}_1, \hat{R}_1, \{\hat{N}_1, \hat{T}_{15}, \hat{A}_{24}\}, \quad (2)$$

$$\hat{F}_{10^*}^c, \hat{F}_{10^*}^{\prime c}, \hat{G}_5^c, \hat{G}_5^{\prime c}, \hat{H}_{u5^*}^c, \hat{H}_{d5}^c, \hat{S}_1^c, \hat{R}_1^c, \{\hat{N}_1^c, \hat{T}_{15^*}^c, \hat{A}_{24}^c\}, \quad (3)$$

where the bold subscripts stand for the representations in $SU(5)$. Since a pair of the fields in the curly brackets $\{\hat{N}_1, \hat{T}_{15}, \hat{A}_{24}\}$ and $\{\hat{N}_1^c, \hat{T}_{15^*}^c, \hat{A}_{24}^c\}$ are necessary to generate nonzero neutrino masses, we introduce one pair of them and in Sec. 6 we will see which fields are compatible with the $SU(1,1)$ weight assignment constrained by other requirements, such as to generate three chiral generations of quarks and leptons and one chiral generations of higgses, to allow Yukawa couplings between quarks and leptons and higgses. The quantum numbers of $SU(5) \times SU(1,1)$ and R -parity are summarized in Table 1. We define the values q_α and q_β as

$$q_\alpha := \alpha' - \alpha, \quad q_\beta := \beta' - \beta, \quad (4)$$

where α, β , etc. are $SU(1,1)$ weights. We choose the values q_α and q_β to be positive half-integers. See Ref. [83] in detail for the notation and convention.

We also introduce the structure fields

$$\hat{\Phi}_1, \hat{\Phi}'_{24}, \hat{\Psi}_{1/24}, \quad (5)$$

where the $SU(1,1)$ spins of $\hat{\Phi}_1$, $\hat{\Phi}'_{24}$, and $\hat{\Psi}_{1/24}$ are S , S' , and S'' , respectively. This is summarized in Table 2. The subscript of $\hat{\Psi}_{1/24}$ represents two options of the $SU(5)$ representations.

We assume that the gauge group $SU(5) \times SU(1,1)$ is spontaneously broken to G_{SM} via the following nonvanishing VEVs of the structure fields

$$\langle \hat{\Phi}_1 \rangle = \langle \phi_0 \rangle, \quad \langle \hat{\Phi}'_{24} \rangle = \langle \phi'_{+1} \rangle, \quad \langle \hat{\Psi}_{1/24} \rangle = \langle \psi_{-3/2} \rangle, \quad (6)$$

where the subscripts of $\langle \phi_0 \rangle$, $\langle \phi'_{+1} \rangle$ and $\langle \psi_{-3/2} \rangle$ stand for the eigenvalues of the third component generator of $SU(1,1)$. In the next section, we will find that the $SU(1,1)$ spins must satisfy $S = S' < S''$, and to realize three generations of quarks and leptons and one generations of higgses, the minimal choice is $S = S' = 1$ and $S'' = 3/2$. We will also find that the VEV of $\hat{\Phi}_{24}$ plays essential roles for decomposing the doublet and triplet higgses and making difference between the Yukawa coupling constants of the down-type quarks and charged leptons. The Clebsch-Gordan coefficients (CGCs) of $SU(5)$ is shown in Ref. [35, 48]. The CGCs of $SU(1,1)$ is found in Ref. [83].

We describe other assumptions as follow. The gauge kinetic function of the $SU(1,1)$ vector superfield and the Kähler potential of the structure fields have positive definite metrics at a vacuum. The Lagrangian in the matter field sector including the coupling terms between matter

Field	$\hat{\Phi}_1$	$\hat{\Phi}'_{24}$	$\hat{\Psi}_{1/24}$
$SU(5)$	1	24	1 or 24
$SU(1,1)$	S	S'	S''
(R -parity)	+	+	+

Table 2: The quantum numbers of structure fields in the $SU(5) \times SU(1,1)$ model are given in the table.

fields and structure fields contains only renormalizable terms, and non-renormalizable terms in superpotential are induced by the process of decoupling the heavy fields. The correction for the Kähler potential of matter fields and the gauge kinetic function of the $SU(5)$ gauge fields is negligible. After the chiral fields are generated via the spontaneous generation of generations [76, 79, 81, 83], the effect from the $SU(1,1)$ gauge bosons and the structure fields is negligible for the chiral matter fields at low energy. Only the structure fields have large VEVs and the matter fields have smaller VEVs compared to those of the structure fields because of maintaining the structures of the horizontal symmetry; e.g., the VEVs of the structure fields are GUT-scale mass $M_{\text{GUT}} \simeq O(10^{16})$ GeV and the VEVs of the matter fields are $m_{\text{SUSY}} \simeq O(10^3)$ GeV. Some $SU(1,1)$ singlet superfields break SUSY in a hidden sector, SUSY breaking does not affect $SU(1,1)$ symmetry, and soft SUSY breaking terms for matter fields are generated at GUT scale $M_{\text{GUT}} \sim O(10^{16})$ GeV in a visible sector, where the soft SUSY breaking masses are $O(m_{\text{SUSY}}) \sim O(10^3)$ GeV.

The number of soft SUSY breaking terms are determined by the number of the superpotential terms. It is impossible to give explicit forms of the soft SUSY braking terms before we discuss the superpotential. Here we just mention the pattern of the soft SUSY breaking terms. Each of trilinear scalar term, so-called A -term, is exactly proportional to the corresponding Yukawa coupling term in the superpotential. Soft scalar masses are generation-independent.

3 Realization of chiral generations

In this section, we consider how to provide three chiral generations of quarks and leptons, one chiral generations of up- and down-type doublet higgses, and no chiral generations of the others shown in Table 1 by using three structure fields shown in Table 2. We use the methods developed in Ref. [83].

3.1 Three chiral generations of quarks and leptons

We discuss how to produce three chiral generations of quarks and leptons. The superpotential of the quark and lepton superfields coupling to the structure fields is given by

$$\begin{aligned}
W = & M_f \hat{F}_{10} \hat{F}_{10}^c + M'_f \hat{F}'_{10} \hat{F}_{10}^c + x_f \hat{F}_{10} \hat{F}_{10}^c \hat{\Phi}_1 + x'_f \hat{F}'_{10} \hat{F}_{10}^c \hat{\Phi}_1 \\
& + z_f \hat{F}_{10} \hat{F}_{10}^c \hat{\Phi}'_{24} + z'_f \hat{F}'_{10} \hat{F}_{10}^c \hat{\Phi}'_{24} + w_f \hat{F}_{10} \hat{F}_{10}^c \hat{\Psi}_{1/24} + w'_f \hat{F}'_{10} \hat{F}_{10}^c \hat{\Psi}_{1/24} \\
& + M_g \hat{G}_{5^*} \hat{G}_5^c + M'_g \hat{G}'_{5^*} \hat{G}_5^c + x_g \hat{G}_{5^*} \hat{G}_5^c \hat{\Phi}_1 + x'_g \hat{G}'_{5^*} \hat{G}_5^c \hat{\Phi}_1 \\
& + z_g \hat{G}_{5^*} \hat{G}_5^c \hat{\Phi}'_{24} + z'_g \hat{G}'_{5^*} \hat{G}_5^c \hat{\Phi}'_{24} + w_g \hat{G}_{5^*} \hat{G}_5^c \hat{\Psi}_{1/24} + w'_g \hat{G}'_{5^*} \hat{G}_5^c \hat{\Psi}_{1/24},
\end{aligned} \tag{7}$$

where M s are mass parameters, x s, z s, and w s are dimensionless coupling constants. We assume that the massless chiral fields are realized as linear combination of the components of \hat{F}_{10} , \hat{F}'_{10} , \hat{G}_{5^*} , and \hat{G}'_{5^*} in manner

$$\hat{f}_{\alpha+i} = \sum_{n=0}^2 \hat{f}_n U_{n,i}^f + [\text{massive modes}], \quad \hat{f}'_{\alpha'+i} = \sum_{n=0}^2 \hat{f}_n U_{n,i}^{f'} + [\text{massive modes}], \tag{8}$$

where \hat{F}_{10} and \hat{F}'_{10} contain \hat{Q} , \hat{U}^c , and \hat{E}^c , and \hat{G}_{5*} and \hat{G}'_{5*} contain \hat{D}^c and \hat{L} . f stands for q , u^c , e^c , d^c , and ℓ . For $f = d^c$, ℓ , α should be replaced by β .

We solve the massless condition by using the mass term of the superpotential in Eq. (7) for the matter field \hat{F}_{10} . For this calculation, there is no difference between the matter fields \hat{F}_{10} and \hat{G}_{5*} except the coupling constants and some CGCs. By substituting the nonvanishing VEVs of the structure fields in Eq. (6) into the superpotential term in Eq. (7), we have the mass term

$$\begin{aligned}
W|_{\Phi=\langle\Phi\rangle} &= M_f \hat{F} \hat{F}^c + M'_f \hat{F}' \hat{F}'^c + x_f \hat{F} \hat{F}^c \langle \hat{\Phi}_1 \rangle + x'_f \hat{F}' \hat{F}'^c \langle \hat{\Phi}_1 \rangle \\
&\quad + Y_f z_f \hat{F} \hat{F}^c \langle \hat{\Phi}'_{24} \rangle + Y_f z'_f \hat{F}' \hat{F}'^c \langle \hat{\Phi}'_{24} \rangle + \tilde{Y}_f w_f \hat{F} \hat{F}^c \langle \hat{\Psi}_{1/24} \rangle + \tilde{Y}_f w'_f \hat{F}' \hat{F}'^c \langle \hat{\Psi}_{1/24} \rangle \\
&= \sum_{i=0}^{\infty} \left[\left(M_f (-1)^i + x_f \langle \phi_0 \rangle D_{i,i}^{\alpha,\alpha,S} \right) \hat{f}_{\alpha+i} \hat{f}_{-\alpha-i}^c + \left(M'_f (-1)^i + x'_f \langle \phi_0 \rangle \right) D_{i,i}^{\alpha',\alpha',S} \hat{f}'_{\alpha'+i} \hat{f}'_{-\alpha'-i}^c \right. \\
&\quad + Y_f z_f \langle \phi'_{+1} \rangle D_{i+1,i}^{\alpha,\alpha,S'} \hat{f}_{\alpha+i} \hat{f}_{-\alpha-i-1}^c + Y_f z'_f \langle \phi'_{+1} \rangle D_{i+1,i}^{\alpha',\alpha',S'} \hat{f}'_{\alpha'+i} \hat{f}'_{-\alpha'-i-1}^c \\
&\quad \left. + \tilde{Y}_f w_f \langle \psi_{-3/2} \rangle D_{i,i+3/2-q_\alpha}^{\alpha,\alpha,S''} \hat{f}'_{\alpha'+i+3/2-q_\alpha} \hat{f}_{-\alpha-i}^c + \tilde{Y}_f w'_f \langle \psi_{-3/2} \rangle D_{i,i+3/2+q_\alpha}^{\alpha',\alpha',S''} \hat{f}_{\alpha+i+3/2+q_\alpha} \hat{f}'_{-\alpha'-i}^c \right] \\
&= \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \left[\hat{f}_n \left\{ \left(M_f (-1)^i + x_f \langle \phi_0 \rangle D_{i,i}^{\alpha,\alpha,S} \right) U_{n,i}^f + Y_f z_f \langle \phi'_{+1} \rangle D_{i,i-1}^{\alpha,\alpha,S'} U_{n,i-1}^f \right. \right. \\
&\quad + \tilde{Y}_f w_f \langle \psi_{-3/2} \rangle D_{i,i+3/2-q_\alpha}^{\alpha,\alpha,S''} U_{n,i+3/2-q_\alpha}^{f'} \left. \right\} \hat{f}_{-\alpha-i}^c \\
&\quad + \hat{f}_n \left\{ \left(M'_f (-1)^i + x'_f \langle \phi_0 \rangle D_{i,i}^{\alpha',\alpha',S} \right) U_{n,i}^{f'} + Y_f z'_f \langle \phi'_{+1} \rangle D_{i+1,i}^{\alpha',\alpha',S'} U_{n,i-1}^{f'} \right. \\
&\quad \left. \left. + \tilde{Y}_f w'_f \langle \psi_{-3/2} \rangle D_{i,i+3/2+q_\alpha}^{\alpha',\alpha',S''} U_{n,i+3/2+q_\alpha}^f \right\} \hat{f}_{-\alpha'-i}^c \right] + [\text{massive modes}], \tag{9}
\end{aligned}$$

where Y_f is a $U(1)_Y$ charge shown in Table 3, and \tilde{Y}_f is equal to one for $\hat{\Psi}_1$ and is equal to Y_f for $\hat{\Psi}_{24}$. The emergence of the massless modes \hat{f}_n requires that the coupling between the massless modes \hat{f}_n and the massive modes $\hat{f}_{-\alpha-i}^c$ and $\hat{f}_{-\alpha'-i}^c$ for any i must vanish simultaneously:

$$X_i U_{n,i}^f + Y_i U_{n,i-1}^f + Z_i U_{n,i+3/2-q_\alpha}^{f'} = 0, \tag{10}$$

$$X'_i U_{n,i}^{f'} + Y'_i U_{n,i-1}^{f'} + Z'_i U_{n,i+3/2+q_\alpha}^f = 0, \tag{11}$$

where we defined the following

$$\begin{aligned}
X_i &:= M_f (-1)^i + x_f \langle \phi_0 \rangle D_{i,i}^{\alpha,\alpha,S}, \quad Y_i := Y_f z_f \langle \phi'_{+1} \rangle D_{i,i-1}^{\alpha,\alpha,S'}, \quad Z_i := \tilde{Y}_f w_f \langle \psi_{-3/2} \rangle D_{i,i+3/2-q_\alpha}^{\alpha,\alpha,S''}, \\
X'_i &:= M'_f (-1)^i + x'_f \langle \phi_0 \rangle D_{i,i}^{\alpha',\alpha',S}, \quad Y'_i := Y_f z'_f \langle \phi'_{+1} \rangle D_{i+1,i}^{\alpha',\alpha',S'}, \quad Z'_i := \tilde{Y}_f w'_f \langle \psi_{-3/2} \rangle D_{i,i+3/2+q_\alpha}^{\alpha',\alpha',S''}.
\end{aligned} \tag{12}$$

These lead to the relation among the mixing coefficients $U_{n,i}^f$ and $U_{n,i}^{f'}$, respectively:

$$U_{n,i+3}^f = \frac{X'_{i+3/2-q_\alpha}}{Z'_{i+3/2-q_\alpha}} \frac{X_i}{Z_i} U_{n,i}^f + \left(\frac{X'_{i+3/2-q_\alpha}}{Z'_{i+3/2-q_\alpha}} \frac{Y_i}{Z_i} + \frac{Y'_{i+3/2-q_\alpha}}{Z'_{i+3/2-q_\alpha}} \frac{X_{i-1}}{Z_{i-1}} \right) U_{n,i-1}^f + \frac{Y'_{i+3/2-q_\alpha}}{Z'_{i+3/2-q_\alpha}} \frac{Y_{i-1}}{Z_{i-1}} U_{n,i-2}^f, \tag{13}$$

$$U_{n,i+3}^{f'} = \frac{X'_{i+3/2+q_\alpha}}{Z'_{i+3/2+q_\alpha}} \frac{X'_i}{Z'_i} U_{n,i}^{f'} + \left(\frac{X_{i+3/2+q_\alpha}}{Z_{i+3/2+q_\alpha}} \frac{Y'_i}{Z'_i} + \frac{Y_{i+3/2+q_\alpha}}{Z_{i+3/2+q_\alpha}} \frac{X'_{i-1}}{Z'_{i-1}} \right) U_{n,i-1}^{f'} + \frac{Y_{i+3/2+q_\alpha}}{Z_{i+3/2+q_\alpha}} \frac{Y'_{i-1}}{Z'_{i-1}} U_{n,i-2}^{f'}, \tag{14}$$

where for $i < 0$, $U_{n,i}^f = U_{n,i}^{f'} = 0$.

Field	$\hat{Q}^{(\prime)}$	$\hat{U}^{c(\prime)}$	$\hat{D}^{c(\prime)}$	$\hat{L}^{(\prime)}$	$\hat{E}^{c(\prime)}$	\hat{H}_u	\hat{H}_d	\hat{T}_u	\hat{T}_d
$SU(3)_C$	3	3*	3*	1	1	1	1	3	3*
$SU(2)_L$	2	1	1	2	1	2	2	1	1
$U(1)_Y$	+1/6	-2/3	+1/3	-1/2	+1	-1/2	+1/2	+1/3	-1/3

Table 3: The quantum numbers of $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ for matter fields in the $SU(5) \times SU(1,1)$ model are given in the table, and their conjugate fields have opposite representations.

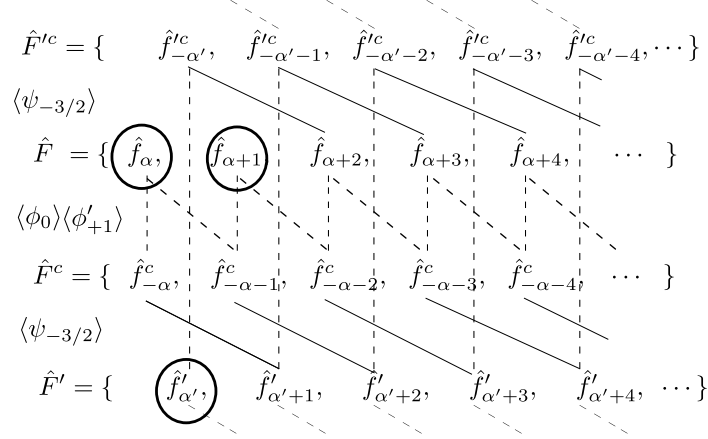


Figure 1: Type-I $q_\alpha = 1/2$ for the three massless generation of quarks and leptons in Eq. (9): a massless mode \hat{f}_0 is realized as certain linear combinations of the components $\hat{f}_{\alpha+3k}$ and $\hat{f}'_{\alpha'+1+3k}$ ($k = 1, 2, \dots$), and the constructional element of the massless mode is determined by its mixing coefficients $U_{0,3k}^f$ and $U_{0,3k+1}^{f'}$ given in Eqs. (13) and (14) and Type-I initial condition. A main element of each massless chiral mode is surround by a circle, and the components of the matter fields \hat{F} , \hat{F}^c , \hat{F}' and \hat{F}'^c connected by solid and dashed lines have a mass term that comes from the VEVs $\langle \psi_{-3/2} \rangle$ and $\langle \phi_0 \rangle$, respectively, and the mass term of the solid line that comes from the VEV $\langle \psi_{-3/2} \rangle$ dominantly contributes to whether massless modes appear or not. The figure wraps from bottom to top.

The recursion equations determine the mixing coefficients $U_{n,i}^f$ and $U_{n,i}^{f'}$ for any i . The two initial condition sets of the mixing coefficients $U_{n,i}^f$ and $U_{n,i}^{f'}$ are also dependent each other. As in Ref. [83], Sec. 3.2, the relation between two initial condition sets can be classified into three conditions: Type-I, $q_\alpha < 3/2$ shown in Fig. 1: Type-II, $q_\alpha = 3/2$ shown in Fig. 2: and Type-III, $q_\alpha > 3/2$ shown in Fig. 3. Each condition leads to the different mixing coefficients, and when we calculate the Yukawa coupling constants, we need to have their detail information. In this paper, we will not analyze the Yukawa coupling in detail. We will just show the difference between the Yukawa couplings of down-type quarks and charged leptons in Sec. 4.

We need to consider the normalizable condition of the mixing coefficients $U_{n,i}^f$ and $U_{n,j}^{f'}$. As in Ref. [83], Sec. 3.2, when the $SU(1,1)$ spins satisfy the condition $S'' > S, S'$, their normalizable conditions are always satisfied regardless of the coupling constants and the value of the VEVs. Thus, three massless modes \hat{f}_n ($n = 0, 1, 2$) appear at low energy. Note that when we make Figs. 1, 2 and 3, we have already assume the $S'' > S, S'$.

One may notice that if the quark and lepton superfield $\hat{G}_{5*}^{(\prime)}$ and the conjugate superfield of up-type higgs \hat{H}_{u5*}^c mix with each other, then this discussion is ruined. Thus, if this model does not include explicitly e.g. the R -parity shown in Table 1 and 2, the $SU(1,1)$ weight content must satisfy $\gamma \neq \beta + [\text{integer or half-integer}]$. In the model with R -parity shown in Table 1 and 2, the quantum number of the quark and lepton superfield $\hat{G}_{5*}^{(\prime)}$ is different from the conjugate

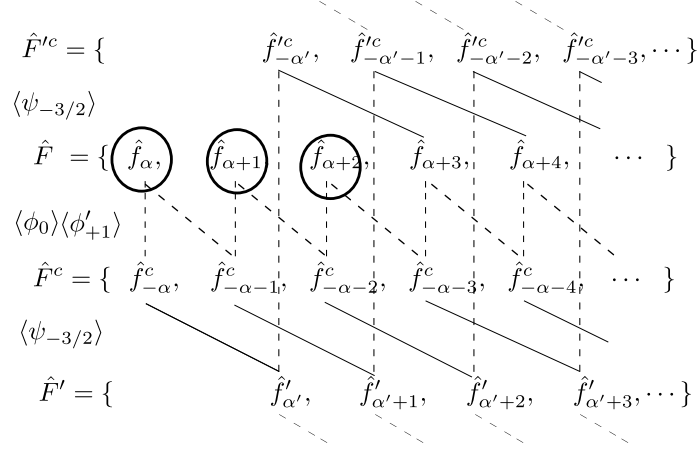


Figure 2: Type-II $q_\alpha = 3/2$ for the three massless generation of quarks and leptons in Eq. (9): each massless mode \hat{f}_n is realized as a certain linear combination of the components $\hat{f}_{\alpha+3k+n}$ and $\hat{f}'_{\alpha'+3k+n}$ ($k = 1, 2, \dots$), and the constructional element of its massless mode is determined by its mixing coefficients $U_{n,3k+n}^f$ and $U_{n,3k+n}^{f'}$ given in Eqs. (13) and (14) and Type-II initial condition. The explanation of the circle and lines is given in Fig. 1.

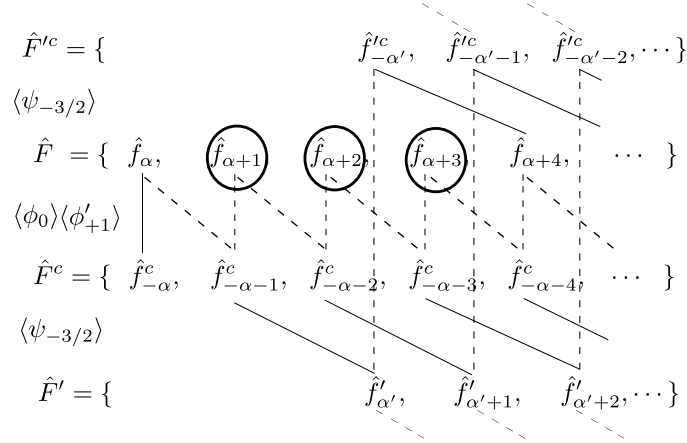


Figure 3: An example of Type-III $q_\alpha = 5/2$ for the three massless generation of quarks and leptons in Eq. (9): each massless mode \hat{f}_n is realized as a certain linear combination of the components $\hat{f}_{\alpha+3k+n+1}$ and $\hat{f}'_{\alpha'+3k+n}$ ($k = 1, 2, \dots$), and the constructional element of its massless mode is determined by its mixing coefficients $U_{n,3k+n+1}^f$ and $U_{n,3k+n}^{f'}$ given in Eqs. (13) and (14) and Type-III initial condition. The explanation of the circle and lines is given in Fig. 1.

field of up-type higgs $\hat{H}_{u\mathbf{5}^*}^c$, so there is no restriction from the part.

3.2 One chiral generation of higgses

We will see that the model can allow only one generation of up- and down-type $SU(2)_L$ doublet higgses and prohibit any generation of the up- and down-type $SU(3)_C$ triplet higgses so-called colored higgs at low energy without fine-tuning and unnatural parameter choice in the sense of 't Hooft naturalness [1]. This is pointed out in Refs. [79, 81].

We consider how to provide one chiral generations of higgses. The superpotential for the matter and structure coupling is

$$W = M_{h_u} \hat{H}_{u\mathbf{5}} \hat{H}_{u\mathbf{5}^*}^c + x_{h_u} \hat{H}_{u\mathbf{5}} \hat{H}_{u\mathbf{5}^*}^c \hat{\Phi}_1 + z_{h_u} \hat{H}_{u\mathbf{5}} \hat{H}_{u\mathbf{5}^*}^c \hat{\Phi}'_{24} \\ + M_{h_d} \hat{H}_{d\mathbf{5}} \hat{H}_{d\mathbf{5}^*}^c + x_{h_d} \hat{H}_{d\mathbf{5}} \hat{H}_{d\mathbf{5}^*}^c \hat{\Phi}_1 + z_{h_d} \hat{H}_{d\mathbf{5}} \hat{H}_{d\mathbf{5}^*}^c \hat{\Phi}'_{24}, \quad (15)$$

where M s are mass parameters, x s and z s are dimensionless coupling constants. We assume that one massless chiral generation of higgses is realized at low energy as a linear combination of the components of $\hat{H}_{u\mathbf{5}}$ and $\hat{H}_{d\mathbf{5}^*}$ in manner

$$\hat{h}_{-\gamma-i} = \hat{h} U_i^h + [\text{massive modes}], \quad (16)$$

where $\hat{H}_{u\mathbf{5}}$ contains \hat{H}_u and \hat{T}_u . $\hat{H}_{d\mathbf{5}^*}$ contains \hat{H}_d and \hat{T}_d . h represents h_u , h_d , t_u , and t_d . For $h = h_d$ and t_d , γ must be replaced by δ .

We solve the massless condition by using the mass term of the superpotential in Eq. (15). The same as the quarks and lepton, we use generic notation \hat{H} for the higgs fields. By substituting the nonvanishing VEVs of the structure fields in Eq. (6) into the superpotential term in Eq. (15), we have the mass term

$$W|_{\Phi=\langle\Phi\rangle} = M_h \hat{H} \hat{H}^c + x_h \hat{H} \hat{H}^c \langle\hat{\Phi}_1\rangle + Y_h z_h \hat{H} \hat{H}^c \langle\hat{\Phi}'_{24}\rangle \\ = \sum_{i=0}^{\infty} \left[\left(M_h (-1)^i + x_h \langle\phi_0\rangle D_{i,i}^{\gamma,\gamma,S} \right) \hat{h}_{-\gamma-i} \hat{h}_{\gamma+i}^c + Y_h z_h \langle\phi'_{+1}\rangle D_{i+1,i}^{\gamma,\gamma,S'} \hat{h}_{-\gamma-i-1} \hat{h}_{\gamma+i}^c \right] \\ = \sum_{i=0}^{\infty} \hat{h} \left\{ \left(M_h (-1)^i + x_h \langle\phi_0\rangle D_{i,i}^{\gamma,\gamma,S} \right) U_{n,i}^h + Y_h z_h \langle\phi'_{+1}\rangle D_{i+1,i}^{\gamma,\gamma,S'} U_{n,i+1}^h \right\} \hat{h}_{\gamma+i}^c \\ + [\text{massive modes}], \quad (17)$$

where Y_h is a $U(1)_Y$ charge shown in Table 3.

The massless mode \hat{h} is extracted from the component $\hat{h}_{-\gamma-i}$ of the matter field \hat{H} . The orthogonality of the massless modes \hat{h} to the massive modes $\hat{h}_{\gamma+i}^c$ requires the coefficients U_i^f to satisfy the following recursion equation for any $i(\geq 0)$

$$\left(M_h (-1)^i + x_h \langle\phi_0\rangle D_{i,i}^{\gamma,\gamma,S} \right) U_{n,i}^h + Y_h z_h \langle\phi'_{+1}\rangle D_{i+1,i}^{\gamma,\gamma,S'} U_{n,i+1}^h = 0. \quad (18)$$

The relation of the mixing coefficients between i th and $i+1$ th components is

$$U_{i+1}^h = - \frac{M_h (-1)^i + x_h \langle\phi_0\rangle D_{i,i}^{\gamma,\gamma,S}}{Y_h z_h \langle\phi'_{+1}\rangle D_{i+1,i}^{\gamma,\gamma,S'}} U_i^h. \quad (19)$$

As is discussed in Ref. [83], Sec. 3.1, we need to consider a normalizable condition $\sum_{i=0}^{\infty} |U_i^h| < \infty$. For $S = S'$, this leads to constraints for the values of the parameters and the nonvanishing VEVs, where we will not consider the $SU(1,1)$ spins satisfying $S < S'$ and $S > S'$ because the condition $S < S'$ provide one chiral doublet and colored higgses and the condition $S > S'$

cannot produce anything at low energy. By using the property of the CGC $D_{i,j}^{\gamma,\gamma,S}$, for the large i limit Eq. (19) becomes

$$\frac{U_{i+1}^h}{U_i^h} \sim \frac{1}{Y_h} \frac{x_h \langle \phi_0 \rangle}{z_h \langle \phi'_{+1} \rangle} \sqrt{\frac{(S+1)!(S-1)!}{S!S!}}, \quad (20)$$

where we dropped the irrelevant term. To satisfy the normalizable condition $\sum_{i=0}^{\infty} |U_i^h| < \infty$, the U_{i+1}^h/U_i^h in Eq. (20) must be smaller than one. When $U_{i+1}^h/U_i^h > 1$, the chiral matter disappears at low energy.

By using the above normalizable condition, we consider the condition to realize existence of the up- and down-type doublet higgses and absence of the up- and down-type colored higgses at low energies. To produce the up- and down-type higgses at low energies, the parameters ϵ_{h_u} and ϵ_{h_d} defined by

$$\epsilon_{h_u} := -2 \frac{x_{h_u} \langle \phi_0 \rangle}{z_{h_u} \langle \phi'_{+1} \rangle}, \quad \epsilon_{h_d} := 2 \frac{x_{h_d} \langle \phi_0 \rangle}{z_{h_d} \langle \phi'_{+1} \rangle} \quad (21)$$

must satisfy the following conditions

$$|\epsilon_{h_u}|, |\epsilon_{h_d}| < \epsilon_{cr}, \quad \epsilon_{cr} := \sqrt{\frac{S!S!}{(S+1)!(S-1)!}}. \quad (22)$$

To eliminate the up- and down-type colored higgses at low energies, the following condition must be satisfied.

$$\epsilon_{cr} < |\epsilon_{t_u}|, |\epsilon_{t_d}|, \quad (23)$$

where the parameters ϵ_{t_u} and ϵ_{t_d} are defined by

$$\epsilon_{t_u} := 3 \frac{x_{h_u} \langle \phi_0 \rangle}{z_{h_u} \langle \phi'_{+1} \rangle}, \quad \epsilon_{t_d} := -3 \frac{x_{h_d} \langle \phi_0 \rangle}{z_{h_d} \langle \phi'_{+1} \rangle}. \quad (24)$$

When we rewrite this condition by using ϵ_{h_u} and ϵ_{h_d} ,

$$\frac{2}{3} \epsilon_{cr} < |\epsilon_{h_u}|, |\epsilon_{h_d}|. \quad (25)$$

Thus, only the up- and down-type higgses appear at low energies if the parameters ϵ_{h_u} and ϵ_{h_d} satisfy the following condition:

$$\frac{2}{3} \epsilon_{cr} < |\epsilon_{h_u}|, |\epsilon_{h_d}| < \epsilon_{cr}. \quad (26)$$

This is shown in Fig. 4

3.3 No chiral generations of others

We consider the $SU(5)$ singlets \hat{S}_1 and \hat{R}_1 with the positive lowest weights η and λ of $SU(1,1)$ and their conjugates

$$W_M = M_s \hat{S}_1 \hat{S}_1^c + M_r \hat{R}_1 \hat{R}_1^c + x_s \hat{S}_1 \hat{S}_1^c \hat{\Phi}_1 + x_r \hat{R}_1 \hat{R}_1^c \hat{\Phi}_1 + z_s \hat{R}_1 \hat{S}_1^c \hat{\Phi}_1 + z'_s \hat{S}_1 \hat{R}_1^c \hat{\Phi}_1, \quad (27)$$

where M_s are mass parameters, x_s and z_s are dimensionless coupling constants. The coupling terms $\hat{R}_1 \hat{S}_1^c \hat{\Phi}_1$ and $\hat{S}_1 \hat{R}_1^c \hat{\Phi}_1$ are allowed if $\Delta_s := \eta - \lambda$ is an integer and the $SU(1,1)$ spin S of the structure field $\hat{\Phi}_1$ is larger or equal to $|\Delta_s|$ ($S \geq |\Delta_s|$). The nonvanishing VEV $\langle \phi_0 \rangle$ of the structure field $\hat{\Phi}_1$ gives additional masses for all components of the singlets. Unfortunately, we need fine-tuning between M_r and $x_r \langle \phi_0 \rangle$ to generate the first components \hat{r} and \hat{r}^c of the

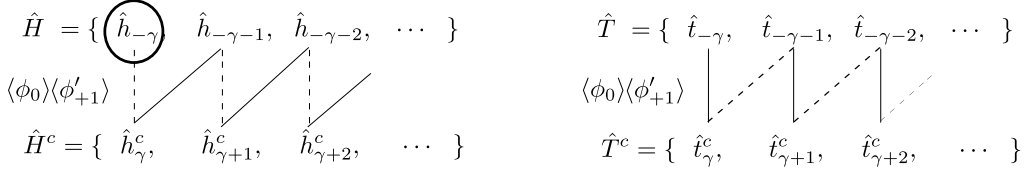


Figure 4: The left-hand figure shows that there is the one massless generation of doublet higgses and each massless mode \hat{h} realized as certain linear combinations of the components $\hat{h}_{-\gamma-k}$ ($k = 1, 2, \dots$), and the constructional element of each massless mode is determined by its mixing coefficients U_k^h in Eq. (19). The right-hand figure shows that there are no massless generations of colored higgses. The explanation of the circle and lines is given in Fig. 1.

Field	\hat{N}	\hat{T}_h	\hat{Q}_h	\hat{C}_h	\hat{A}_ℓ	\hat{W}_ℓ	\hat{G}_ℓ	\hat{X}_ℓ	\hat{Y}_ℓ
$SU(3)_C$	1	1	6	3	1	1	8	3	3*
$SU(2)_L$	1	3	1	2	1	3	1	2	2
$U(1)_Y$	0	+1	-2/3	+1/6	0	0	0	-5/6	+5/6

Table 4: The quantum numbers of $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ for matter fields in the $SU(5) \times SU(1,1)$ model are given in the table, and their conjugate fields have opposite representations. \hat{N} belongs to \hat{N}_1 . \hat{T}_h , \hat{Q}_h , \hat{C}_h belong to \hat{T}_{15} . \hat{A}_ℓ , \hat{W}_ℓ , \hat{G}_ℓ , \hat{X}_ℓ , \hat{Y}_ℓ belong to \hat{A}_{24} .

matter fields \hat{R}_1 and \hat{R}_1^c with the mass $O(m_{\text{SUSY}})$. These light fields are necessary to produce the effective μ -term of up- and down-type higgses $O(m_{\text{SUSY}})$. Note that the other components of \hat{R}_1 and \hat{R}_1^c have at least $O(M_{\text{GUT}})$ because of the difference of the CGC of $SU(1,1)$ between $\hat{R}_1 \hat{R}_1^c$ and $\hat{R}_1 \hat{R}_1^c \hat{\Phi}_1$. In other words, we cannot realize more than one massless vectorlike matter fields at low energy for each matter field. The other singlets \hat{S}_1 and \hat{S}_1^c have the masses at least $O(M_{\text{GUT}})$.

We must emphasize the above fine-tuning problem. This is obviously unnatural, and this unnaturalness strongly suggests the incompleteness of this model. To solve the fine-tuning problem, one may prefer to use tiny mass $M_r \sim O(m_{\text{SUSY}})$ compared to $O(M_{\text{GUT}})$ and tiny dimensionless coupling constant $x_r \sim O(m_{\text{SUSY}}/G_{\text{GUT}})$ compared to $O(1)$ without any reason. Alternatively, models that include additional matter and structure fields may lead to massless SM singlets \hat{r} and \hat{r}^c via spontaneous generation of generations without any naturalness problem. However, we will not pursue this possibility in this paper.

We next discuss the fields that are necessary to generate neutrino masses via seesaw mechanisms. First, we consider $SU(5)$ singlet \hat{N}_1 with the positive lowest weight ξ of $SU(1,1)$ and the conjugate, respectively. The superpotential contains

$$W_N = M_n \hat{N}_1 \hat{N}_1^c + x_n \hat{N}_1 \hat{N}_1^c \hat{\Phi}_1, \quad (28)$$

where M_n is a mass parameter and x_n is a dimensionless coupling constant. The same as the fields \hat{S}_1 and \hat{R}_1 , the nonvanishing VEVs $\langle \phi_0 \rangle$ and $\langle \phi'_{+1} \rangle$ of the structure field $\hat{\Phi}_1$ in Eq. (17) give huge masses to all components of the matter fields \hat{N}_1 and \hat{N}_1^c . Here we assume that the coupling terms such as $\hat{N}_1 \hat{S}_1 \Phi_1$ and $\hat{N}_1 \hat{R}_1 \Phi_1$ are forbidden by R -parity or the $SU(1,1)$ weight conditions. We will discuss in Sec. 6.

Second, we consider $SU(5)$ **15**-plet \hat{T}_{15} with the negative highest weight τ of $SU(1,1)$ and its conjugate because $SU(2)_L$ triplet \hat{T}_h is contained in $SU(5)$ **15**-plet \hat{T}_{15} . The superpotential contains

$$W_N = M_t \hat{T}_{15} \hat{T}_{15}^c + x_t \hat{T}_{15} \hat{T}_{15}^c \hat{\Phi}_1 + z_t \hat{T}_{15} \hat{T}_{15}^c \hat{\Phi}'_{24}, \quad (29)$$

where M_t is a mass parameter, and x_t and z_t are dimensionless coupling constants. The nonvanishing VEVs $\langle\phi_0\rangle$ and $\langle\phi'_{+1}\rangle$ of the structure fields $\hat{\Phi}_{\mathbf{1}}$ and $\hat{\Phi}'_{\mathbf{24}}$ in Eq. (17) give huge masses to all components of the matter fields $\hat{T}_{\mathbf{15}}$ and $\hat{T}_{\mathbf{15}^*}^c$. In this case, the discussion of whether massless particles appear or not is exactly the same as higgs cases. The parameter ϵ_{t_h} defined as

$$\epsilon_{t_h} := \frac{x_t \langle\phi_0\rangle}{z_t \langle\phi'_{+1}\rangle} \quad (30)$$

must satisfy the condition

$$|\epsilon_{t_h}| > \epsilon_{cr} \quad (31)$$

for the triplet higgs \hat{T}_h to disappear at low energy. In this case, the other fields \hat{Q}_h and \hat{C}_h in the $SU(5)$ $\mathbf{15}$ -plet $\hat{T}_{\mathbf{15}}$ shown in Table 4 automatically disappear at low energy because the triplet higgs has the largest $U(1)_Y$ charge within the $SU(5)$ $\mathbf{15}$ -plet.

One may suspect that, if the coupling terms $\hat{F}_{\mathbf{10}} \hat{T}_{\mathbf{15}^*}^c \hat{\Phi}_{\mathbf{24}}$ and $\hat{F}_{\mathbf{10}^*}^c \hat{T}_{\mathbf{15}} \hat{\Phi}_{\mathbf{24}}$ are allowed, they could disturb the structure of the chiral generations for quarks and leptons. Fortunately, both $\hat{F}_{\mathbf{10}}^{(\prime)}$ and $\hat{T}_{\mathbf{15}^*}^c$ belong to positive fields, where positive fields are chiral superfields with the positive weight of $SU(1,1)$. Thus these couplings are not allowed.

Third, we consider $SU(5)$ $\mathbf{24}$ -plet $\hat{A}_{\mathbf{24}}$ with the positive lowest weight ζ of $SU(1,1)$ and the conjugate, respectively. The superpotential contains

$$W_N = M_a \hat{A}_{\mathbf{24}} \hat{A}_{\mathbf{24}}^c + x_a \hat{A}_{\mathbf{24}} \hat{A}_{\mathbf{24}}^c \hat{\Phi}_{\mathbf{1}} + z_a \hat{A}_{\mathbf{24}} \hat{A}_{\mathbf{24}}^c \hat{\Phi}'_{\mathbf{24}}, \quad (32)$$

where M_a is a mass parameter, and x_a and z_a are dimensionless coupling constants. All components of the matter fields \hat{A}_ℓ , \hat{W}_ℓ , \hat{G}_ℓ shown in Table 4 and their conjugate fields automatically have masses via the nonvanishing VEVs $\langle\phi_0\rangle$ and $\langle\phi'_{+1}\rangle$ of the structure fields $\hat{\Phi}_{\mathbf{1}}$ and $\hat{\Phi}'_{\mathbf{24}}$ in Eq. (17) because their $U(1)_Y$ charges are zero. The other components of the fields \hat{X}_ℓ , \hat{Y}_ℓ and their conjugate fields are massive when the parameters ϵ_x and ϵ_y defined as

$$\epsilon_x := -\frac{6}{5} \frac{x_a \langle\phi_0\rangle}{z_a \langle\phi'_{+1}\rangle}, \quad \epsilon_y := \frac{6}{5} \frac{x_a \langle\phi_0\rangle}{z_a \langle\phi'_{+1}\rangle} \quad (33)$$

satisfy the following condition

$$|\epsilon_x| = |\epsilon_y| > \epsilon_{cr}. \quad (34)$$

4 Structures of Yukawa couplings

We now discuss the Yukawa couplings between quarks and leptons and higgses.

$$W_Y = y_{\mathbf{10}} \hat{F}_{\mathbf{10}}^{(\prime)} \hat{F}_{\mathbf{10}}^{(\prime)} \hat{H}_{u\mathbf{5}} + y_{\mathbf{5}} \hat{F}_{\mathbf{10}}^{(\prime)} \hat{G}_{\mathbf{5}^*}^{(\prime)} \hat{H}_{d\mathbf{5}^*}, \quad (35)$$

where $y_{\mathbf{10}}$ and $y_{\mathbf{5}}$ are dimensionless coupling constants. Each Yukawa coupling can be classified into two types. For the first term in Eq. (35) of $\hat{F}_{\mathbf{10}}^{(\prime)}$ and $\hat{H}_{u\mathbf{5}}$, one is $\gamma = 2\alpha + [\text{positive half-integer}]$;

$$W_Y = y_{\mathbf{10}} \hat{F}_{\mathbf{10}} \hat{F}'_{\mathbf{10}} \hat{H}_{u\mathbf{5}}, \quad (36)$$

where $y_{\mathbf{10}}$ is a coupling constant; the other is $\gamma = 2\alpha + [\text{semi-positive integer}]$.

$$W_Y = y_{\mathbf{10}} \hat{F}_{\mathbf{10}} \hat{F}_{\mathbf{10}} \hat{H}_{u\mathbf{5}} + y'_{\mathbf{10}} \hat{F}'_{\mathbf{10}} \hat{F}'_{\mathbf{10}} \hat{H}_{u\mathbf{5}}, \quad (37)$$

where y'_{10} is a coupling constant, and the second term is allowed if $\gamma \geq 2\alpha'$. For the second term in Eq. (35), one is $\gamma = 2\alpha + [\text{positive half-integer}]$;

$$W_Y = y_5 \hat{F}'_{10} \hat{G}_{5^*} \hat{H}_{d5^*} + y'_5 \hat{F}_{10} \hat{G}'_{5^*} \hat{H}_{d5^*}, \quad (38)$$

where y_5 and y'_5 are coupling constants, the first term is allowed if $\delta \geq \alpha' + \beta$ and the second term is allowed if $\delta \geq \alpha + \beta'$; the other is $\delta = \alpha + \beta + [\text{semi-positive integer}]$.

$$W_Y = y_5 \hat{F}_{10} \hat{G}_{5^*} \hat{H}_{d5^*} + y'_5 \hat{F}'_{10} \hat{G}'_{5^*} \hat{H}_{d5^*}, \quad (39)$$

where the second term is allowed if $\delta \geq \alpha' + \beta'$.

We see the structure of the Yukawa couplings in Eqs. (37)–(38). After we extract massless modes, we can generally write the superpotential terms of the Yukawa coupling constant at the low energy

$$W = y_u^{mn} \hat{q}_m \hat{u}_n^c \hat{h}_u + y_d^{mn} \hat{q}_m \hat{d}_n^c \hat{h}_d + y_e^{mn} \hat{\ell}_m \hat{e}_n^c \hat{h}_d. \quad (40)$$

For the superpotential in Eq. (37), the Yukawa coupling constants of up-type quarks are

$$y_u^{mn} = \sum_{i,j=0}^{\infty} \left(y_{10} C_{i,j}^{\alpha,\alpha,\Delta_\alpha} U_{m,i}^q U_{n,j}^u U_{i+j}^{h_u} + y'_{10} C_{i,j}^{\alpha',\alpha',\Delta'_\alpha} U_{m,i}^{q'} U_{n,j}^{u'} U_{i+j}^{h_u} \right), \quad (41)$$

where $C_{i,j}^{\alpha,\alpha,\Delta_\alpha}$ is a CGC of $SU(1,1)$ given in Ref. [83], U s are given by the spontaneous generation of generations discussed in Sec. 3, $\Delta_\alpha := \gamma - 2\alpha$ and $\Delta'_\alpha := \gamma - 2\alpha'$. For the superpotential in Eq. (36), the Yukawa coupling constants of up-type quarks are

$$y_u^{mn} = \sum_{i,j=0}^{\infty} y_{10} \left(C_{i,j}^{\alpha,\alpha',\Delta_\alpha} U_{m,i}^q U_{n,j}^{u'} + C_{i,j}^{\alpha',\alpha,\Delta_\alpha} U_{m,i}^{q'} U_{n,j}^u \right) U_{i+j}^{h_u}. \quad (42)$$

For the superpotential in Eq. (39), the Yukawa coupling constants of down-type quarks and charged leptons are

$$y_d^{mn} = \sum_{i,j=0}^{\infty} \left(y_5 C_{i,j}^{\alpha,\beta,\Delta_\beta} U_{m,i}^q U_{n,j}^d + y'_5 C_{i,j}^{\alpha',\beta,\Delta'_\beta} U_{m,i}^{q'} U_{n,j}^{d'} \right) U_{i+j}^{h_d}, \quad (43)$$

$$y_e^{mn} = \sum_{i,j=0}^{\infty} \left(y_5 C_{i,j}^{\beta,\alpha,\Delta_\beta} U_{m,i}^\ell U_{n,j}^e + y'_5 C_{i,j}^{\beta',\alpha',\Delta'_\beta} U_{m,i}^{\ell'} U_{n,j}^{e'} \right) U_{i+j}^{h_d}. \quad (44)$$

For the superpotential in Eq. (38), the Yukawa coupling constants of down-type quarks and charged leptons are

$$y_d^{mn} = \sum_{i,j=0}^{\infty} \left(y_5 C_{i,j}^{\alpha',\beta,\Delta_\beta} U_{m,i}^{q'} U_{n,j}^d + y'_5 C_{i,j}^{\alpha,\beta',\Delta'_\beta} U_{m,i}^q U_{n,j}^{d'} \right) U_{i+j}^{h_d}, \quad (45)$$

$$y_e^{mn} = \sum_{i,j=0}^{\infty} \left(y_5 C_{i,j}^{\beta,\alpha',\Delta_\beta} U_{m,i}^\ell U_{n,j}^{e'} + y'_5 C_{i,j}^{\beta',\alpha,\Delta'_\beta} U_{m,i}^{\ell'} U_{n,j}^e \right) U_{i+j}^{h_d}. \quad (46)$$

The Yukawa coupling constants are completely determined by the overall couplings y s and the mixing coefficients U s. In particular, the weight condition satisfies $\gamma = \alpha + \alpha'$, $\delta = \alpha' + \beta$, and $q_\alpha < q_\beta$. Each Yukawa coupling matrix has only one overall coupling constant.

As we have already discussed in Sec. 3, the mixing coefficients of down-type quarks and charged leptons in Eqs. (13) and (14) contain the $SU(5)$ breaking contributions, so the Yukawa coupling constants of down-type quarks can be different from those of charged leptons.

The patterns of the mixing coefficients are highly dependent on the values of q_α and q_β that determine dominant massless components. The detail investigation of the Yukawa couplings is not the purpose in this paper, so we will not analyze the mass eigenvalues of quarks and leptons, and the CKM [93, 94] and MNS [95] matrices. One can find the basic argument in Refs. [76, 79, 81, 83].

5 μ -term

We need to generate the effective μ -term $\mu \hat{h}_u \hat{h}_d$, where $\mu \simeq O(m_{\text{SUSY}})$ is the supersymmetry breaking mass parameter $O(10^{2\sim 3})$ GeV [82]. This is because the μ -term $\mu \hat{H}_u \hat{H}_d$ is forbidden by the noncompact horizontal symmetry $G_N = SU(1, 1)$ since both chiral higgses \hat{h}_u and \hat{h}_d are contained in the negative fields \hat{H}_u and \hat{H}_d [76, 96], where negative fields are chiral superfields with the negative weight of $SU(1, 1)$. To generate the effective μ -term, the up- and down-type higgses \hat{H}_u and \hat{H}_d must couple to a positive field \hat{S} belonging to the singlet under G_{SM} , and the field must get the nonvanishing VEV $O(m_{\text{SUSY}})$. Unlike the Next-to-minimal supersymmetric SM (NMSSM) that contains an extra singlet superfield under G_{SM} , the horizontal symmetry does not allow the existence of linear, quadratic and cubic terms, e.g., $M^2 \hat{S}$, $M \hat{S}^2$, and $\lambda \hat{S}^3$. Thus, in this model, we cannot use the same method as in the NMSSM.

If the up- and down-type higgses \hat{h}_u and \hat{h}_d belong to conjugate representations or the same real representation, then the effective μ -term $\mu \hat{h}_u \hat{h}_d$ is generated only by singlet fields and the VEVs of the intermediate scale $O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$ between the supersymmetry breaking mass scale $O(m_{\text{SUSY}})$ and the fundamental scale $O(M_{\text{GUT}})$. If the up- and down-type higgses do not belong to conjugate representations, then the effective μ -term needs to be generated not only by singlet representations of compact unified group but also non-singlet representations. The seesaw mechanism between the fundamental scale $O(M_{\text{GUT}})$ and the intermediate scale $O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$ generates the supersymmetry breaking mass scale $O(m_{\text{SUSY}})$.

Let us first consider how to generate the non-vanishing VEVs with the supersymmetry breaking mass scale $O(m_{\text{SUSY}})$ and the intermediate scale $O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$ from the fundamental scale $O(M_{\text{GUT}})$ and the supersymmetry breaking mass scale $O(m_{\text{SUSY}})$. To realize this situation, we need to introduce some matter fields with singlet under $SU(5)$.

The simplest superpotential contains the $SU(5)$ singlets \hat{S}_1 and \hat{R}_1 with the positive lowest weights $\eta = \gamma + \delta$ and $\lambda = (\gamma + \delta)/2$ of $SU(1, 1)$ and their conjugates

$$W_M = y \hat{R}_1 \hat{R}_1 \hat{S}_1^c + y^c \hat{R}_1^c \hat{R}_1^c \hat{S}_1 + y' \hat{H}_{u5} \hat{H}_{d5^*} \hat{S}_1 + y'^c \hat{H}_{u5^*}^c \hat{H}_{d5}^c \hat{S}_1^c, \quad (47)$$

where y s are coupling constants. From the above superpotential and the superpotential in Eq. (27), decoupling the singlets except the first component of the singlets \hat{S}_1 , \hat{S}_1^c , \hat{R}_1 and \hat{R}_1^c , we obtain

$$W_M = \tilde{M}_s \hat{s} \hat{s}^c + \tilde{M}_r \hat{r} \hat{r}^c + y \hat{r} \hat{r}^c \hat{s} + y^c \hat{r}^c \hat{r}^c \hat{s} + y' U_0^{h_u} U_0^{h_d} \hat{h}_{u5} \hat{h}_{d5^*} \hat{s}, \quad (48)$$

where we assume $\tilde{M}_s := M_s + x_s \langle \phi_0 \rangle \sim O(M_{\text{GUT}})$ and $\tilde{M}_r := M_r + x_r \langle \phi_0 \rangle \sim O(m_{\text{SUSY}})$, the $U_0^{h_u}$ and $U_0^{h_d}$ are mixing coefficients of the up- and down-type higgses. Decoupling \hat{s} and \hat{s}^c by using

$$\frac{\partial W}{\partial \hat{s}} = \tilde{M}_s \hat{s}^c + y^c \hat{r}^c \hat{r}^c + y' U_0^{h_u} U_0^{h_d} \hat{h}_{u5} \hat{h}_{d5^*} = 0, \quad \frac{\partial W}{\partial \hat{s}^c} = \tilde{M}_s \hat{s} + y \hat{r} \hat{r} = 0, \quad (49)$$

we have

$$W_M = \tilde{M}_r \hat{r} \hat{r}^c - \frac{y}{\tilde{M}_s} \hat{r} \hat{r}^c \left(y^c \hat{r}^c \hat{r}^c + y' U_0^{h_u} U_0^{h_d} \hat{h}_{u5} \hat{h}_{d5^*} \right). \quad (50)$$

This leads to the scalar potential

$$V_{\text{SUSY}} = \left| \frac{\partial W}{\partial r} \right|^2 + \left| \frac{\partial W}{\partial r^c} \right|^2 = \left| \tilde{M}_r r^c - 2 \frac{y y^c}{\tilde{M}_s} r r^c r^c \right|^2 + \left| \tilde{M}_r r - 2 \frac{y y^c}{\tilde{M}_s} r r r^c \right|^2. \quad (51)$$

Its corresponding SUSY breaking terms are

$$V_{\text{SUSY}} = B_r \tilde{M}_r r r^c - A_r \frac{y y^c}{\tilde{M}_s} r r r^c r^c + \text{h.c.} + \tilde{m}_r^2 |r|^2 + \tilde{m}_r^2 |r^c|^2, \quad (52)$$

where B_r is a B -parameter of \hat{r} and \hat{r}^c , A_r is an A -parameter of $\hat{r}\hat{r}^c$, and \tilde{m}_r^2 and \tilde{m}_r^{c2} are soft masses of \hat{r} and \hat{r}^c , respectively. The total scalar potential is

$$V = V_{\text{SUSY}} + V_{\text{SUSY}}. \quad (53)$$

After we perform tedious calculation, we obtain $\langle r \rangle, \langle r^c \rangle = O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$ and $\langle s \rangle = O(m_{\text{SUSY}})$ as discussed in Ref. [82]. Thus, the effective μ -term between h_{u5} and h_{d5} is $O(m_{\text{SUSY}})$. The singlet fermions and scalars \hat{r} and \hat{r}^c have a mass term $O(m_{\text{SUSY}})$ except the Nambu-Goldstone (NG) boson since this potential have a $U(1)$ global symmetry at low energy and this symmetry is broken by the nonvanishing VEVs of the singlets. Note that, if there is no SUSY breaking term, the singlet fermion is massless because SUSY forces the fermionic partner of the NG boson to be a pseudo-NG fermion [61, 66, 97, 98].

In addition, the coupling between the higgses \hat{h}_{u5} and \hat{h}_{d5^*} and the singlets is suppressed by the factor $O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$. Therefore, the effective theory below the energy scale $\sqrt{m_{\text{SUSY}} M_{\text{GUT}}}$ is described by the MSSM and the almost decoupled G_{SM} singlets.

6 Baryon and/or lepton number violating terms

We classify the baryon and/or lepton number violating terms up to superpotential dimension-4 by using $SU(1,1)$ symmetry and the R -parity [84] (matter parity [85]) shown in Table 1. For a review, see, e.g., Ref. [86]. In the following, we omit the mirror terms. λ s stand for a dimensionless coupling, Λ s and μ are dimension-one parameters, and Δ s are integer.

First, $SU(1,1)$ symmetry and R -parity allow the following \mathcal{B} and/or \mathcal{L} dimension-4 operator

$$W_{M:4;\mathcal{B},\mathcal{L}} = \frac{1}{\Lambda} \hat{G}_{5^*}^{(\prime)} \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} \hat{H}_{u5} = \frac{1}{\Lambda} \hat{L}^{(\prime)} \hat{L}^{(\prime)} \hat{H}_u \hat{H}_u + \frac{1}{\Lambda} \hat{D}^{c(\prime)} \hat{D}^{c(\prime)} \hat{T}_u \hat{T}_u + \frac{1}{\Lambda} \hat{D}^{c(\prime)} \hat{L}^{(\prime)} \hat{T}_u \hat{H}_u \quad (54)$$

if the $SU(1,1)$ weights satisfy a condition $\Delta = 2\gamma - \beta^{(\prime)} - \beta^{(\prime)}$. More explicitly, when $\Delta = 2\gamma - 2\beta$, $\hat{G}_{5^*} \hat{G}_{5^*} \hat{H}_{u5} \hat{H}_{u5}$ are allowed; when $\Delta = 2\gamma - \beta - \beta'$, $\hat{G}_{5^*} \hat{G}_{5^*}' \hat{H}_{u5} \hat{H}_{u5}$ are allowed; when $\Delta = 2\gamma - 2\beta'$, $\hat{G}_{5^*}' \hat{G}_{5^*}' \hat{H}_{u5} \hat{H}_{u5}$ are allowed. In the following we also use the same rule.

Second, $SU(1,1)$ symmetry prohibits and R -parity allows the following \mathcal{B} and/or \mathcal{L} dimension-4 operator

$$W_{M:4;\mathcal{B},\mathcal{L}} = \frac{1}{\Lambda} \hat{F}_{10}^{(\prime)} \hat{F}_{10}^{(\prime)} \hat{F}_{10}^{(\prime)} \hat{G}_{5^*}^{(\prime)} = \frac{1}{\Lambda} \hat{Q}^{(\prime)} \hat{Q}^{(\prime)} \hat{Q}^{(\prime)} \hat{L}^{(\prime)} + \frac{1}{\Lambda} \hat{U}^{(\prime)c} \hat{U}^{(\prime)c} \hat{D}^{(\prime)c} \hat{E}^{(\prime)c} \quad (55)$$

because $\hat{F}_{10}^{(\prime)}$ and $\hat{G}_{5^*}^{(\prime)}$ belong to positive fields.

Third, $SU(1,1)$ symmetry and R -parity prohibit the following \mathcal{B} and/or \mathcal{L} dimension-3 operator

$$W_{M:3;\mathcal{B},\mathcal{L}} = \lambda \hat{F}_{10}^{(\prime)} \hat{G}_{5^*}^{(\prime)} \hat{G}_{5^*}^{(\prime)} = \lambda \hat{E}^{(\prime)c} \hat{L}^{(\prime)} \hat{L}^{(\prime)} + \lambda \hat{Q}^{(\prime)} \hat{D}^{(\prime)c} \hat{L}^{(\prime)} + \lambda \hat{U}^{(\prime)c} \hat{D}^{(\prime)c} \hat{D}^{(\prime)c} \quad (56)$$

because $\hat{F}_{10}^{(\prime)}$ and $\hat{G}_{5^*}^{(\prime)}$ belong to positive fields.

Finally, $SU(1,1)$ symmetry allows and R -parity prohibits the following \mathcal{B} and/or \mathcal{L} dimension-2 operator

$$W_{M:2;\mathcal{B},\mathcal{L}} = \mu \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} = \mu \hat{L}^{(\prime)} \hat{H}_u + \mu \hat{D}^{(\prime)c} \hat{T}_u \quad (57)$$

if $\beta^{(\prime)} = \gamma$. The dimension-3 operators are

$$\begin{aligned} W_{M:3;\mathcal{B},\mathcal{L}} &= \lambda \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} \hat{S}_1 + \lambda' \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} \hat{S}_1^c + \lambda'' \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} \hat{R}_1 + \lambda''' \hat{G}_{5^*}^{(\prime)} \hat{H}_{u5} \hat{R}_1^c \\ &= \lambda \hat{L}^{(\prime)} \hat{H}_u \hat{S} + \lambda \hat{D}^{(\prime)} \hat{T}_u \hat{S} + \lambda' \hat{L}^{(\prime)} \hat{H}_u \hat{S}^c + \lambda' \hat{D}^{(\prime)} \hat{T}_u \hat{S}^c \\ &\quad + \lambda'' \hat{L}^{(\prime)} \hat{H}_u \hat{R} + \lambda'' \hat{D}^{(\prime)} \hat{T}_u \hat{R} + \lambda''' \hat{L}^{(\prime)} \hat{H}_u \hat{R}^c + \lambda''' \hat{D}^{(\prime)} \hat{T}_u \hat{R}^c, \end{aligned} \quad (58)$$

if $\gamma = \eta + \beta^{(\prime)} + \Delta$, $\beta^{(\prime)} = \eta + \gamma + \Delta$, $\gamma = \lambda + \beta^{(\prime)} + \Delta$, and $\beta^{(\prime)} = \lambda + \gamma + \Delta$;

$$W_{M:3;\mathcal{B};\mathcal{L}} = \lambda \hat{F}_{10}^{(\prime)} \hat{H}_{d5^*} \hat{H}_{d5^*} + \lambda' \hat{F}_{10}^{(\prime)} \hat{F}_{10}^{(\prime)} \hat{G}_5^{(\prime)c} \\ = \lambda \hat{E}^{(\prime)c} \hat{H}_d \hat{H}_d + \lambda \hat{Q}^{(\prime)} \hat{T}_d \hat{H}_d + \lambda \hat{U}^{(\prime)c} \hat{T}_d \hat{T}_d + \lambda' \hat{Q}^{(\prime)} \hat{Q}^{(\prime)} \hat{D}^{(\prime)} + \lambda' \hat{Q}^{(\prime)} \hat{U}^{(\prime)c} \hat{L}^{(\prime)c}, \quad (59)$$

if $\alpha^{(\prime)} = 2\gamma + \Delta$ and $\beta^{(\prime)} = \alpha^{(\prime)} + \alpha^{(\prime)} + \Delta$. The dimension-4 operators are

$$W_{M:4;\mathcal{B};\mathcal{L}} = \frac{1}{\Lambda} \hat{F}_{10}^{(\prime)} \hat{G}_5^{(\prime)} \hat{G}_5^{(\prime)} \hat{S}_1^c + \frac{1}{\Lambda'} \hat{F}_{10}^{(\prime)} \hat{G}_5^{(\prime)} \hat{G}_5^{(\prime)} \hat{R}_1^c + \frac{1}{\Lambda''} \hat{F}_{10}^{(\prime)} \hat{F}_{10}^{(\prime)} \hat{F}_{10}^{(\prime)} \hat{H}_{d5^*} + \frac{1}{\Lambda'''} \hat{G}_5^{(\prime)} \hat{H}_{d5^*} \hat{H}_{u5} \hat{H}_{u5} \\ = \frac{1}{\Lambda} \hat{E}^{(\prime)c} \hat{L}^{(\prime)} \hat{L}^{(\prime)} \hat{S}^c + \frac{1}{\Lambda} \hat{Q}^{(\prime)} \hat{D}^{(\prime)c} \hat{L}^{(\prime)} \hat{S}^c + \frac{1}{\Lambda} \hat{U}^{(\prime)c} \hat{D}^{(\prime)c} \hat{D}^{(\prime)c} \hat{S}^c \\ + \frac{1}{\Lambda'} \hat{E}^{(\prime)c} \hat{L}^{(\prime)} \hat{L}^{(\prime)} \hat{R}^c + \frac{1}{\Lambda'} \hat{Q}^{(\prime)} \hat{D}^{(\prime)c} \hat{L}^{(\prime)} \hat{R}^c + \frac{1}{\Lambda'} \hat{U}^{(\prime)c} \hat{D}^{(\prime)c} \hat{D}^{(\prime)c} \hat{R}^c \\ + \frac{1}{\Lambda''} \hat{Q}^{(\prime)} \hat{Q}^{(\prime)} \hat{Q}^{(\prime)} \hat{H}_d + \frac{1}{\Lambda''} \hat{Q}^{(\prime)} \hat{U}^{(\prime)c} \hat{E}^{(\prime)c} \hat{H}_d \\ + \frac{1}{\Lambda'''} \hat{L}^{(\prime)} \hat{H}_d \hat{H}_u \hat{H}_u + \frac{1}{\Lambda'''} \hat{D}^{(\prime)c} \hat{T}_d \hat{T}_u \hat{T}_u + \frac{1}{\Lambda'''} \hat{L}^{(\prime)} \hat{T}_d \hat{H}_u \hat{T}_u \quad (60)$$

if $\lambda = \alpha^{(\prime)} + \alpha^{(\prime)} + \beta^{(\prime)} + \Delta$, $\eta = \alpha^{(\prime)} + \alpha^{(\prime)} + \beta^{(\prime)} + \Delta$, $\delta = \alpha^{(\prime)} + \alpha^{(\prime)} + \alpha^{(\prime)}$, and $\beta^{(\prime)} = 2\gamma + \delta + \Delta$.

In general, SUSY models with R -parity violating terms suffer from rapid proton decay and lepton flavor violations [86]. Thus, to prevent the unacceptable predictions, the R -parity must be realized at low energy. Fortunately, even when we discuss SUSY models with R -parity that contain the relevant or marginal terms, after some heavy particles are integrated out, the effective neutrino “mass” term in Eq. (54) can be induced. Unfortunately, a problematic operator in Eq. (55) can be also induced.

On the other hand, the $SU(1, 1)$ horizontal symmetry does not allow the problematic operator in Eq. (55). Of course, Once the symmetry is broken, there is no reason to deny generating the operator. We will discuss this topic in this section.

Another interesting feature is that special weight assignments of $SU(1, 1)$ mean that R -parity remains even after the $SU(1, 1)$ symmetry is broken. One assignment is the following:

$$\alpha = \frac{2n+1}{4}, \quad \alpha' = \frac{2n+1}{4} + q_\alpha, \quad \beta = \frac{2n+1}{4} + 2m, \quad \beta' = \frac{2n+1}{4} + 2m + q_\beta, \\ \gamma = n + q_\alpha + \frac{1}{2}, \quad \delta = n + 2m + q_\alpha + \frac{1}{2}, \quad \eta/2 = \lambda = n + m + q_\alpha + \frac{1}{2}, \quad (61)$$

where the $SU(1, 1)$ weight, such as α , must be a positive number, n and m are integer, q_α and q_β are half-integer. In other words, the quark and lepton superfields have the quarter values of the $SU(1, 1)$ weight, and the higgs and the other superfields have integer values of the $SU(1, 1)$ weight. Thus, even numbers of quarks and leptons are necessary to couple higgses and the other fields. This is completely the same as the R -parity shown in Table 1. When we construct models with an $SU(1, 1)$ horizontal symmetry, we do not always assume the R -parity to prevent rapid proton decay, lepton flavor violation and to make dark matter candidate. Note that the assignment is compatible with the Yukawa couplings in Eqs. (36) and (38), but incompatible with those in Eqs. (37) and (39).

Also, another example is the following assignment

$$\alpha = \frac{2n+1}{4}, \quad \alpha' = \frac{2n+1}{4} + q_\alpha, \quad \beta = \frac{2n-3}{4} + 2m - 3q_\alpha - q_\beta, \quad \beta' = \frac{2n-3}{4} + 2m - 3q_\alpha, \\ \gamma = n + 2q_\alpha + \frac{1}{2}, \quad \delta = n + 2m - 2q_\alpha - \frac{1}{2}, \quad \eta/2 = \lambda = n + m, \quad (62)$$

where n, m are integer. This assignment is compatible with those in Eqs. (37) and (39), but incompatible with the Yukawa couplings in Eqs. (36) and (38). The same as the assignment in Eq. (61), this assignment forbids all R -parity violating terms because the quarks and leptons

have the quarter values of the $SU(1, 1)$ weight, the higgses have the half-integer values, and the singlets have the integer values.

We can find other assignments of $SU(1, 1)$ weights to prohibit R -parity violating terms, and to allow the “neutrino mass” term in Eq. (54). The above two assignments in Eq. (61) and Eq. (62) include enough assignments for the following discussion. We only consider the model with these $SU(1, 1)$ assignment or explicitly imposed R -parity shown in Table 1. We focus on the superpotential terms in Eqs. (54) and (55). We will discuss how to obtain sizable neutrino masses and how to suppress rapid proton decay in this model.

6.1 Neutrino masses

We now discuss seesaw mechanisms, so-called Type-I [6, 99], Type-II [90–92], and Type-III [100] seesaw mechanisms in the MSSM plus additional necessary field context. Type-I, Type-II, and Type-III seesaw mechanisms can be achieved by using right-handed neutrinos \hat{N} with $(\mathbf{1}, \mathbf{1}, 0)$ under G_{SM} , charged triplet higgses \hat{T}_h and \hat{T}_h^c with $(\mathbf{1}, \mathbf{3}, \pm 1)$ under G_{SM} , and neutral triplet leptons \hat{W}_ℓ with $(\mathbf{1}, \mathbf{3}, 0)$, where the first, second and third columns stand for an $SU(3)_C$ weight, an $SU(2)_L$ weight, and a $U(1)_Y$ charge, respectively. We can also classify Type-I and Type-II seesaw mechanisms as Majorana-type seesaw mechanism and Type-II as non-Majorana-type. For a review, see, e.g., Ref. [101].

Each additional field has the following superpotential terms, respectively

$$W_I = \sum_{a,b} M_N^{ab} \hat{N}_a \hat{N}_b + \sum_{i,a} y_I^{ia} \hat{L}_i \hat{H}_u \hat{N}_a, \quad (63)$$

$$W_{II} = \sum_{a,b} M_{T_h}^{ab} \hat{T}_{ha} \hat{T}_{hb}^c + \sum_{i,j,a} y_{II}^{ija} \hat{L}_i \hat{L}_j \hat{T}_{ha} + \sum_a y_{II}^{ia} \hat{H}_u \hat{H}_u \hat{T}_{ha}, \quad (64)$$

$$W_{III} = \sum_{a,b} M_{W_\ell}^{ab} \hat{W}_{\ell a} \hat{W}_{\ell b} + \sum_{i,a} y_{III}^{ia} \hat{L}_i \hat{H}_u \hat{W}_{\ell a}, \quad (65)$$

where M s are mass parameters, y s are coupling constant, a, b stand for the label of the additional matter fields, and i is the label of the left-handed neutrino. If we assume M_X is much larger than electro-weak scale, after decoupling the additional fields, we obtain the effective neutrino-higgs superpotential term

$$W_{\text{eff}} = \sum_{i,j=1}^3 \frac{\kappa^{ij}}{M_X} \hat{L}_i \hat{L}_j \hat{H}_u \hat{H}_u, \quad (66)$$

where M_X is a mass parameter and κ^{ij} is a coupling constant matrix determined by the mass parameters and the coupling constants in Eq. (63)–(65). After the up-type higgs \hat{H}_u obtains a non-vanishing VEV, the coupling term becomes the mass term of the left-handed neutrinos. If M_X is $O(M_{\text{GUT}})$, the effective masses become $O(v_{\text{EW}}^2/M_{\text{GUT}}) \sim O(10^{-3})$ eV. The current experimental data for neutrino masses is $\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 = 0.00232_{-0.00008}^{+0.00012} \text{ eV}^2$ [102], so the mediated particles seems to be better to have smaller mass $O(10^{14}) - O(10^{15})$ GeV than GUT-scale mass $M_{\text{GUT}} \sim O(10^{16})$ GeV.

For $SU(5)$ GUT models, Type-I, Type-II, and Type-III seesaw mechanisms can be also achieved by using $SU(5)$ singlet fields \hat{N}_1 $SU(5)$ $\mathbf{15}$ -plet and $\mathbf{15}^*$ -plet fields $\hat{T}_{\mathbf{15}}$ and $\hat{T}_{\mathbf{15}^*}^c$, and $SU(5)$ $\mathbf{24}$ -plet fields $\hat{A}_{\mathbf{24}}$, respectively. Note that since $\hat{A}_{\mathbf{24}}$ contains \hat{A}_ℓ and \hat{W}_ℓ , this field includes not only Type-I seesaw but also Type-III seesaw mechanisms.

We move on to our $SU(5) \times SU(1, 1)$ model. As we have already seen before, the Majorana mass terms are not allowed by the $SU(1, 1)$ symmetry. One may think that the Type-I and Type-III seesaw mechanisms are prohibited, but as we discussed for the effective μ term of the up- and down-type higgs doublets, once the horizontal symmetry is broken, there is no reason to prohibit the Majorana mass terms.

We discuss two situations Majorana-type Type-I and Type-III seesaw mechanisms and Dirac-type Type-I, Type-II, and Type-III seesaw mechanisms for the massive mediated superfields realized by the spontaneous generations of generations discussed in Sec. 3.

We start by considering the Majorana-type Type-I and Type-III seesaw mechanisms. The masses of the mediated fields come from the Dirac mass term of the fields and their conjugate fields, and the masses are different from the Majorana masses μ_X , where μ_X stands for the Majorana mass of \hat{N}_1 or \hat{A}_{24} . Our basic assumption is that μ_X is much smaller than the Dirac mass of the mediated fields. When we integrate out the mediated fields, we obtain the effective neutrino masses $O(\mu_X v_{\text{EW}}^2/M_{\text{GUT}}^2)$. If we assume $\mu_N \sim O(\sqrt{m_{\text{SUSY}} M_{\text{GUT}}})$, $O(\mu_N v_{\text{EW}}^2/M_{\text{GUT}}^2) \sim O(10^{-10})$ eV. This is too tiny. Therefore, the Majorana-type seesaw mechanisms cannot explain the observed neutrino masses.

Next we discuss Dirac-type Type-I, Type-II, and Type-III seesaw mechanisms. The superpotential terms are given by

$$W_{\text{D-I}} = y_I \hat{G}_{5^*} \hat{H}_{u5} \hat{N}_1 + y_I^c \hat{G}_{5^*}' \hat{H}_{u5} \hat{N}_1^c, \quad (67)$$

$$W_{\text{D-II}} = y_{II} \hat{G}_{5^*} \hat{G}_{5^*}' \hat{T}_{15} + y_{II}^c \hat{H}_{u5} \hat{H}_{u5} \hat{T}_{15}^c, \quad (68)$$

$$W_{\text{D-III}} = y_{III} \hat{G}_{5^*} \hat{H}_{u5} \hat{A}_{24} + y_{III}^c \hat{G}_{5^*}' \hat{H}_{u5} \hat{A}_{24}^c, \quad (69)$$

where $\gamma = \beta + \xi + \Delta_\xi$, $\beta' = \gamma + \xi + \Delta'_\xi$, $\tau = \beta + \beta' + \Delta_\tau$, $\tau = 2\gamma + \Delta'_\tau$, $\gamma = \beta + \zeta + \Delta_\zeta$, and $\beta' = \gamma + \zeta + \Delta'_\zeta$. Δ s are non-negative integer. To realize the seesaw mechanisms, we have to choose the $SU(1,1)$ weight assignment satisfying the following condition:

$$2\gamma = \beta + \beta' + \Delta_x - \Delta'_x, \quad (70)$$

where x stands for ξ , τ , or ζ . This leads to a constraint

$$n = 4m + \Delta_x - \Delta'_x - 2q_\alpha + q_\beta - \frac{1}{2} \quad (71)$$

for the $SU(1,1)$ weight assignment in Eq. (61), and that also leads to a constraint

$$n = 4m + \Delta_x - \Delta'_x - 10q_\alpha - q_\beta - \frac{5}{2} \quad (72)$$

for the $SU(1,1)$ weight assignment in Eq. (62). After decoupling the heavy matter, we obtain the effective superpotential

$$W_N = \sum_{n,m=0}^2 \kappa_{n,m}^X \hat{\ell}_n \hat{\ell}_m \hat{h}_u \hat{h}_u, \quad (73)$$

where $\kappa_{n,m}^X$ are coupling constants, and X stands for I, II, and III,

$$\kappa_{n,m}^I := \sum_{i,j,k=0}^{\infty} -\frac{y_I y_I^c}{\tilde{M}_j^I} C_{i,j}^{\beta,\xi,\Delta_\xi} C_{k,j}^{\gamma,\xi,\Delta'_\xi} U_{n,i}^\ell U_{m,k+j-\Delta'_\xi}^{\ell'} U_{i+j-\Delta_\xi}^{h_u} U_k^{h_u}, \quad (74)$$

$$\kappa_{n,m}^{II} := \sum_{i,j,k=0}^{\infty} -\frac{y_{II} y_{II}^c}{\tilde{M}_j^{II}} C_{i,j+j-\Delta_\tau}^{\beta,\beta',\Delta_\tau} C_{k,j-k+\Delta'_\tau}^{\gamma,\gamma,\Delta'_\tau} U_{n,i}^\ell U_{m,j-i-\Delta_\tau}^{\ell'} U_k^{h_u} U_{j-k+\Delta'_\tau}^{h_u}, \quad (75)$$

$$\kappa_{n,m}^{III} := \sum_{i,j,k=0}^{\infty} -\frac{y_{III} y_{III}^c}{\tilde{M}_j^{III}} C_{i,j}^{\beta,\zeta,\Delta_\zeta} C_{k,j}^{\gamma,\zeta,\Delta'_\zeta} U_{n,i}^\ell U_{m,k+j-\Delta'_\zeta}^{\ell'} U_{i+j-\Delta_\zeta}^{h_u} U_k^{h_u}, \quad (76)$$

where \tilde{M}_j^X is the mass of the j th component of the Type-I, II, III. The VEV $\langle h_u \rangle = O(v_{\text{EW}})$ of the chiral up-type higgs \hat{h}_u generates the effective neutrino mass matrix $m_\nu^{mn} \hat{\ell}_m \hat{\ell}_n$ ($m, n = 0, 1, 2$) with $m_\nu^{mn} = O(v_{\text{EW}}^2/M_{\text{GUT}})$.

6.2 Proton decay

Before we discuss proton decay in our model, we quickly review the proton decay discussion in the minimal $SU(5)$ SUSY GUT [53–56]. First, the superpotential of the Yukawa couplings in models with the minimal $SU(5)$ matter content contains the following baryon and/or lepton number violation terms

$$W = y_{10} \hat{F}_{10} \hat{F}_{10} \hat{H}_{u5} \ni y_{10} \hat{Q} \hat{Q} \hat{T}_u + y_{10} \hat{E}^c \hat{U}^c \hat{T}_u, \quad (77)$$

$$W = y_{10} \hat{F}_{10} \hat{G}_{5^*} \hat{H}_{d5^*} \ni y_5 \hat{Q} \hat{L} \hat{T}_d + y_5 \hat{U}^c \hat{D}^c \hat{T}_d. \quad (78)$$

After the doublet part of the original $SU(5)$ μ -term $\mu_5 \hat{H}_{u5} \hat{H}_{d5}$ is canceled by using the “ μ ”-term induced from the VEVs of the coupling between the $SU(5)$ adjoint and up- and down-type higgses $\langle \hat{\Phi}_{24} \rangle \hat{H}_{u5} \hat{H}_{d5^*}$, we can obtain the effective μ parameter of the doublet higgses $\mu \sim O(m_{\text{SUSY}})$ and of the colored higgses $M_C \sim O(M_{\text{GUT}})$. After the colored higgses decouple, they lead to two dimension 4 superpotential terms breaking baryon and/or lepton number

$$W_5 = -\frac{1}{M_C} \sum_{m,n,p,q=0}^2 \left(\frac{1}{2} C_{5L}^{mnpq} \hat{q}_m \hat{q}_n \hat{q}_p \hat{\ell}_q + C_{5R}^{mnpq} \hat{e}_m^c \hat{u}_n^c \hat{u}_p^c \hat{d}_q^c \right), \quad (79)$$

where C_{5X}^{mnpq} ($X = L, R$) are dimensionless coupling constants that depend on the Yukawa coupling matrices of quarks and leptons. According to the analysis discussed in Ref. [56], we use the recent super-Kamiokande result for the lifetime $\tau(p \rightarrow K^+ \bar{\nu}) > 3.3 \times 10^{33}$ years at 90% C.L. [58]. Assuming that soft SUSY breaking parameters at the Planck scale are described by the universal scalar mass, universal gaugino mass, and universal coefficient of the trilinear scalar coupling, so-called A -term and the sfermion mass $m_{\tilde{f}}$ is less than 1 TeV, the colored higgs mass M_C must be larger than 10^{17} GeV for $\tan \beta$ ($2 < \tan \beta < 5$); 10^{18} GeV for $\tan \beta = 10$; 10^{19} GeV for $\tan \beta = 30$; and 10^{20} GeV for $\tan \beta = 50$. (Recently, it was discussed in Ref. [103] that when the sfermion mass is much greater than 1 TeV, the colored higgs mass M_C can be 10^{16} GeV regardless of $\tan \beta$.)

We move on to discuss proton decay in our model. The chiral matter content is realized via the spontaneous generation of generations discussed in Sec. 3. As discussed in Sec. 3.2, once the up- and down-type double higgses appear and the up- and down-type colored higgses disappear at a vacuum, the up- and down-type colored higgses have their Dirac masses, respectively. To generate the baryon and/or lepton number violation terms in Eq. (79), they must include the μ -term between the colored higgses. We discuss two assignments in Eqs. (61) and (62). The effective superpotential is

$$W = \sum_{m,n,p,q=0}^2 \lambda_{m,n,p,q} \hat{q}_m \hat{q}_n \hat{q}_p \hat{\ell}_q + \lambda'_{m,n,p,q} \hat{e}_m^c \hat{u}_n^c \hat{u}_p^c \hat{d}_q^c, \quad (80)$$

where $\lambda_{m,n,p,q}$ and $\lambda'_{m,n,p,q}$ are determined by the colored higgs masses, the μ -parameter of the colored higgses, the overall Yukawa couplings, and the mixing coefficients of quarks and leptons. For the assignment in Eq. (61), the Yukawa couplings in Eqs. (36) and (38) lead to

$$\lambda_{m,n,p,q} = \sum_{i,j,k,\ell=0}^{\infty} \left[y_{10} y_5 \frac{\mu_{i+j,k+\ell}}{M_{t_u i+j} M_{t_d k+\ell}} C_{i,j}^{\alpha,\alpha',0} C_{k,\ell}^{\alpha',\beta,0} U_{m,i}^q U_{n,j}^{q'} U_{p,k}^{q'} U_{q,\ell}^{\ell} \right. \\ \left. + y_{10} y'_5 \frac{\mu_{i+j,k+\ell-q_\beta+q_\alpha}}{M_{t_u i+j} M_{t_d k+\ell-q_\beta+q_\alpha}} C_{i,j}^{\alpha,\alpha',0} C_{k,\ell}^{\alpha,\beta',q_\beta-q_\alpha} U_{m,i}^q U_{n,j}^{q'} U_{p,k}^q U_{q,\ell}^{\ell'} \right], \quad (81)$$

$$\lambda'_{m,n,p,q} = \sum_{i,j,k,\ell=0}^{\infty} \left[y_{10} y_5 \frac{\mu_{i+j,k+\ell}}{M_{t_u i+j} M_{t_d k+\ell}} C_{i,j}^{\alpha,\alpha',0} C_{k,\ell}^{\alpha',\beta,0} U_{m,i}^e U_{n,j}^{u'} U_{p,k}^{u'} U_{q,\ell}^d \right. \\ \left. + y_{10} y'_5 \frac{\mu_{i+j,k+\ell-q_\beta+q_\alpha}}{M_{t_u i+j} M_{t_d k+\ell-q_\beta+q_\alpha}} C_{i,j}^{\alpha,\alpha',0} C_{k,\ell}^{\alpha,\beta',q_\beta-q_\alpha} U_{m,i}^e U_{n,j}^{u'} U_{p,k}^u U_{q,\ell}^{d'} \right], \quad (82)$$

where

$$\mu_{i,j} := \mu \left(\prod_{r=0}^{i-1} \frac{M'_{t_u r}}{M_{t_u r}} \right) \left(\prod_{s=0}^{j-1} \frac{M'_{t_d s}}{M_{t_d s}} \right) \quad (83)$$

and

$$\begin{aligned} M_{t_u i} &:= M_{h_u}(-1)^i + x_{h_u} \langle \phi_0 \rangle D_{i,i}^{\gamma,\gamma,S}, & M'_{t_u i} &:= Y_{t_u} z_{h_u} \langle \phi'_{+1} \rangle D_{i+1,i}^{\gamma,\gamma,S'}, \\ M_{t_d i} &:= M_{h_d}(-1)^i + x_{h_d} \langle \phi_0 \rangle D_{i,i}^{\delta,\delta,S}, & M'_{t_d i} &:= Y_{t_d} z_{h_d} \langle \phi'_{+1} \rangle D_{i+1,i}^{\delta,\delta,S'}. \end{aligned} \quad (84)$$

For the assignment in Eq. (62), the Yukawa couplings in Eqs. (37) and (39) lead to

$$\begin{aligned} \lambda_{m,n,p,q} = \sum_{i,j,k,\ell=0}^{\infty} & \left[y_{10} y_5 \frac{\mu_{i+j-2q_\alpha, k+\ell-q_\alpha-q_\beta}}{M_{t_u i+j-2q_\alpha} M_{t_d k+\ell-q_\alpha-q_\beta}} C_{i,j}^{\alpha,\alpha,2q_\alpha} C_{k,\ell}^{\alpha,\beta,q_\alpha+q_\beta} U_{m,i}^q U_{n,j}^q U_{p,k}^q U_{q,\ell}^\ell \right. \\ & + y_{10} y'_5 \frac{\mu_{i+j-2q_\alpha, k+\ell}}{M_{t_u i+j-2q_\alpha} M_{t_d k+\ell}} C_{i,j}^{\alpha,\alpha,2q_\alpha} C_{k,\ell}^{\alpha',\beta',0} U_{m,i}^q U_{n,j}^q U_{p,k}^{q'} U_{q,\ell}^{\ell'} \\ & + y'_{10} y_5 \frac{\mu_{i+j, k+\ell-q_\alpha-q_\beta}}{M_{t_u i+j} M_{t_d k+\ell-q_\alpha-q_\beta}} C_{i,j}^{\alpha',\alpha',0} C_{k,\ell}^{\alpha,\beta,q_\alpha+q_\beta} U_{m,i}^{q'} U_{n,j}^{q'} U_{p,k}^q U_{q,\ell}^\ell \\ & \left. + y'_{10} y'_5 \frac{\mu_{i+j, k+\ell}}{M_{t_u i+j} M_{t_d k+\ell}} C_{i,j}^{\alpha',\alpha',0} C_{k,\ell}^{\alpha',\beta',0} U_{m,i}^{q'} U_{n,j}^{q'} U_{p,k}^{q'} U_{q,\ell}^{\ell'} \right], \end{aligned} \quad (85)$$

$$\begin{aligned} \lambda'_{m,n,p,q} = \sum_{i,j,k,\ell=0}^{\infty} & \left[y_{10} y_5 \frac{\mu_{i+j-2q_\alpha, k+\ell-q_\alpha-q_\beta}}{M_{t_u i+j-2q_\alpha} M_{t_d k+\ell-q_\alpha-q_\beta}} C_{i,j}^{\alpha,\alpha,2q_\alpha} C_{k,\ell}^{\alpha,\beta,q_\alpha+q_\beta} U_{m,i}^e U_{n,j}^u U_{p,k}^u U_{q,\ell}^d \right. \\ & + y_{10} y'_5 \frac{\mu_{i+j-2q_\alpha, k+\ell}}{M_{t_u i+j-2q_\alpha} M_{t_d k+\ell}} C_{i,j}^{\alpha,\alpha,2q_\alpha} C_{k,\ell}^{\alpha',\beta',0} U_{m,i}^e U_{n,j}^u U_{p,k}^{u'} U_{q,\ell}^{d'} \\ & + y'_{10} y_5 \frac{\mu_{i+j, k+\ell-q_\alpha-q_\beta}}{M_{t_u i+j} M_{t_d k+\ell-q_\alpha-q_\beta}} C_{i,j}^{\alpha',\alpha',0} C_{k,\ell}^{\alpha,\beta,q_\alpha+q_\beta} U_{m,i}^{e'} U_{n,j}^{u'} U_{p,k}^u U_{q,\ell}^d \\ & \left. + y'_{10} y'_5 \frac{\mu_{i+j, k+\ell}}{M_{t_u i+j} M_{t_d k+\ell}} C_{i,j}^{\alpha',\alpha',0} C_{k,\ell}^{\alpha',\beta',0} U_{m,i}^{e'} U_{n,j}^{u'} U_{p,k}^{u'} U_{q,\ell}^{d'} \right]. \end{aligned} \quad (86)$$

Note that as we discussed in Sec. 5, the original μ term between up- and down-type higgses is prohibited by the horizontal symmetry. In the model, the nonvanishing VEVs of the singlets generate the μ -term of the 0th component of \hat{h}_{u5} and \hat{h}_{d5^*} .

We need to consider the experimental bound for proton decay in the model. In the calculation in Ref. [56] it is assumed that the Yukawa coupling matrices of the colored higgses are the same as the matrices of the down-type higgses. In the current model, the Yukawa coupling matrices of the doublet higgses are different from the colored higgses, so we cannot use directly the constraint for the mass of the colored higgses discussed in Ref. [56]. When we assume that the C_{5X}^{mnpq} in Eq. (79) is almost the same as the coupling constant in Eq. (80) normalized by using the effective mass $M_{t_u 0} M_{t_d 0} / \mu_{0,0}$, and we compare Eq. (79) with the above coupling constants, the value of the effective M_C is $M_{t_u 0} M_{t_d 0} / \mu_{0,0}$. When we assume $M_{t_u 0} \sim M_{t_d 0} \sim M_{\text{GUT}} \sim O(10^{16})$ GeV and $\mu_{0,0} \sim m_{\text{SUSY}} \sim O(10^3)$, we obtain $M_C \sim O(10^{29})$ GeV. This value is far from the current colored higgs mass bound $O(10^{17})$ GeV to $O(10^{20})$ GeV. Thus, the proton decay effect caused by the colored higgs is negligible once the colored higgs are massive via the spontaneous generation of generations. The dominant contribution for proton decay modes comes from the X and Y gauge boson exchanges. The dominant proton decay mode $p \rightarrow \pi^0 e^+$ via the X and Y gauge bosons must be found first. In other words, if one of the current or planned near future proton decay experiments finds another proton decay mode, e.g., $p \rightarrow K^+ \bar{\nu}$ before $p \rightarrow \pi^0 e^+$ are found, this model will be excluded.

Finally, we verify the contribution from additional matter fields \hat{C}_h and \hat{Q}_h in the **15**-plet \hat{T}_{15} , and \hat{X}_ℓ and \hat{Y}_ℓ in the **24**-plet \hat{A}_{24} , and their conjugate fields. These operators cannot generate the

superpotential dimension-4 operators in Eq. (55), so the lowest contribution can only come from at least superpotential dimension-5 operators. This means that since the nonvanishing VEVs of the non-SM singlets are those of up- and down-type higgses, the contribution of them for proton decay are suppressed by at least $m_{\text{SUSY}}/M_{\text{GUT}} \sim O(10^{-13})$ compared to the superpotential dimension-4 operators in Eq. (55). Thus, they are completely negligible at least for the current experimental bound.

7 Summary and discussion

We discussed the $SU(5)$ SUSY GUT model with the $SU(1, 1)$ horizontal symmetry that includes the matter fields in Table 1 and the structure fields in Table 2. We showed that the mechanism of the spontaneous generation of generations produces the matter content of the MSSM and the almost decoupled G_{SM} singlets through the nonvanishing VEVs of the structure fields given in Eq. (6). For quarks and leptons, the nonvanishing VEV $\langle\psi_{-3/2}\rangle$ of the structure field $\hat{\Psi}_{1/24}$ with the $SU(1, 1)$ half-integer spin S'' plays the important role for producing the three chiral generations of quarks and leptons. The nonvanishing VEV $\langle\phi'_{+1}\rangle$ of the structure field $\hat{\Phi}'_{24}$ with the $SU(1, 1)$ integer S' leads to the difference between the mixing coefficients of quarks and leptons because the structure field $\hat{\Phi}'_{24}$ belongs to the nontrivial representation of $SU(5)$. Thus, the mixing coefficients of the down-type quarks are different from those of the charged leptons. This avoids the unacceptable prediction in the minimal $SU(5)$ GUT model for the down-type quark's and the charged lepton's Yukawa coupling constants. For higgses, the nonvanishing VEV $\langle\psi_{-3/2}\rangle$ does not affect anything because the structure field $\hat{\Psi}_{1/24}$ does not couple to the higgs superfields. Due to the fact, the nonvanishing VEVs $\langle\phi_0\rangle$ and $\langle\phi'_{+1}\rangle$ of the structure fields $\hat{\Phi}_1$ and $\hat{\Phi}'_{24}$ with the $SU(1, 1)$ integers S and S' determine whether the higgses appear or not. The VEVs can produce only one generation of the up- and down-type doublet higgses at low energy. We found that the model naturally realizes the doublet-triplet mass splitting between the doublet and colored higgses pointed out in Ref. [79, 81].

We also found that some special $SU(1, 1)$ assignments allow only the \mathcal{B} and/or \mathcal{L} superpotential dimension-4 operator $\hat{G}_5^* \hat{G}'_5 \hat{H}_{u5} \hat{H}_{u5}$, which contains the $\hat{L} \hat{L}' \hat{H}_u \hat{H}_u$, up to superpotential dimension-4 operators. The assignments retain R -parity even after the $SU(1, 1)$ symmetry is broken. Thus, we can identify the $SU(1, 1)$ assignments as the origin of the R -parity.

We found that this model can generate the neutrino masses via not only the Type-II seesaw mechanism but also the Type-I and Type-III seesaw mechanisms. We also found that the neutrino masses are dependent on the mixing coefficients of the leptons and up-type higgses, the $SU(1, 1)$ CGCs, the masses of the mediated fields, and their overall Yukawa coupling constants.

We verified that the proton decay via the superpotential dimension-4 operators generated by decoupling the colored higgses is highly suppressed compared to that of usual GUT models. The suppression factor is roughly $O(m_{\text{SUSY}}/M_{\text{GUT}}) \sim O(10^{-13})$. Thus, the dominant contribution to proton decay comes from the X and Y gauge bosons. Thus, the dominant proton decay mode $p \rightarrow \pi^0 e^+$ via the X and Y gauge bosons must be found first. In other words, if another proton decay mode, e.g., $p \rightarrow K^+ \bar{\nu}$, are discovered before $p \rightarrow \pi^0 e^+$ are found, this model will be excluded.

We mention the gauge anomalies of G_{SM} at low energies. The spontaneous generation of generations allows apparent anomalous chiral matter content at low energies because the apparent anomalies should be canceled out by the Wess-Zumino-Witten term [24, 82, 104]. For example, the up-type colored higgs could appear at low energy while the down-type colored higgs disappears at low energy. In this case, the matter content at low energy is anomalous. Of course, since in this situation there is a massless colored higgsino, this is unacceptable. At present, we have no idea to avoid this situation except choosing the vacuum states of the structure fields to produce anomaly-free matter content.

We have not yet solved the vacuum structure in a model that includes at least three structure fields with two $SU(1, 1)$ integer spins S , S' and one $SU(1, 1)$ half-integer spin S'' . To produce

three chiral generations of quarks and leptons and one generation of higgses, the $SU(1, 1)$ spins must satisfy the relation $S'' > S = S' \geq 1$. Thus, the minimal choice is $S = S' = 1$, $S'' = 3/2$. We must discuss the model to justify the assumption of this article.

Acknowledgments

The author would like to thank Jonathan P. Hall, Kenzo Inoue, and Hirofumi Kubo for helpful comments. This work was supported in part by the Department of Energy under grant DE-FG02-91ER40661 and by the Indiana University Center for Spacetime Symmetries.

References

- [1] G. 't Hooft, (ed.) *et al.*, “Recent Developments in Gauge Theories,” *NATO Adv. Study Inst. Ser. B Phys.* **59** (1980) 1–438.
- [2] S. Dimopoulos and H. Georgi, “Softly Broken Supersymmetry and $SU(5)$,” *Nucl.Phys.* **B193** (1981) 150.
- [3] R. Barbieri and G. F. Giudice, “Upper Bounds on Supersymmetric Particle Masses,” *Nucl. Phys.* **B306** (1988) 63.
- [4] F. Wilczek and A. Zee, “Horizontal Interaction and Weak Mixing Angles,” *Phys. Rev. Lett.* **42** (1979) 421.
- [5] C. D. Froggatt and H. B. Nielsen, “Hierarchy of Quark Masses, Cabibbo Angles and CP Violation,” *Nucl. Phys.* **B147** (1979) 277.
- [6] T. Yanagida, “Horizontal Gauge Symmetry and Masses of Neutrinos,”. In Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, p95 (1979).
- [7] T. Maehara and T. Yanagida, “Gauge Symmetry of Horizontal Flavor,” *Prog. Theor. Phys.* **61** (1979) 1434.
- [8] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Y. S. Tyupkin, “Pseudoparticle Solutions of the Yang-Mills Equations,” *Phys. Lett.* **B59** (1975) 85–87.
- [9] G. 't Hooft, “Symmetry Breaking through Bell-Jackiw Anomalies,” *Phys. Rev. Lett.* **37** (1976) 8–11.
- [10] R. Jackiw and C. Rebbi, “Vacuum Periodicity in a Yang-Mills Quantum Theory,” *Phys. Rev. Lett.* **37** (1976) 172–175.
- [11] C. G. Callan, Jr., R. F. Dashen, and D. J. Gross, “The Structure of the Gauge Theory Vacuum,” *Phys. Lett.* **B63** (1976) 334–340.
- [12] A. E. Nelson, “Naturally Weak CP Violation,” *Phys. Lett.* **B136** (1984) 387.
- [13] A. E. Nelson, “Calculation of θ Barr,” *Phys. Lett.* **B143** (1984) 165.
- [14] S. M. Barr, “A Natural Class of NonPeccei-Quinn Models,” *Phys. Rev.* **D30** (1984) 1805.
- [15] R. N. Mohapatra and G. Senjanovic, “Natural Suppression of Strong P and T Noninvariance,” *Phys. Lett.* **B79** (1978) 283–286.
- [16] R. N. Mohapatra and A. Rasin, “Simple Supersymmetric Solution to the Strong CP Problem,” *Phys. Rev. Lett.* **76** (1996) 3490–3493, [arXiv:hep-ph/9511391](#).

- [17] K. S. Babu, B. Dutta, and R. N. Mohapatra, “Solving the Strong CP and the SUSY Phase Problems with Parity Symmetry,” *Phys. Rev.* **D65** (2002) 016005, [arXiv:hep-ph/0107100](#).
- [18] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,” *Phys. Rev. Lett.* **38** (1977) 1440–1443.
- [19] R. D. Peccei and H. R. Quinn, “Constraints Imposed by CP Conservation in the Presence of Instantons,” *Phys. Rev.* **D16** (1977) 1791–1797.
- [20] S. Weinberg, “A New Light Boson?,” *Phys. Rev. Lett.* **40** (1978) 223–226.
- [21] F. Wilczek, “Problem of Strong P and T Invariance in the Presence of Instantons,” *Phys. Rev. Lett.* **40** (1978) 279–282.
- [22] J. E. Kim, “Weak Interaction Singlet and Strong CP Invariance,” *Phys. Rev. Lett.* **43** (1979) 103.
- [23] H.-Y. Cheng, “The Strong CP Problem Revisited,” *Phys. Rept.* **158** (1988) 1.
- [24] J. Wess and B. Zumino, “Consequences of Anomalous Ward Identities,” *Phys. Lett.* **B37** (1971) 95.
- [25] J. Wess and J. Bagger, “Supersymmetry and Supergravity,”. Princeton, USA: Univ. Pr. (1992) 259 p.
- [26] H. P. Nilles, “Supersymmetry, Supergravity and Particle Physics,” *Phys. Rept.* **110** (1984) 1–162.
- [27] H. Hatanaka, T. Inami, and C. S. Lim, “The Gauge Hierarchy Problem and Higher Dimensional Gauge Theories,” *Mod. Phys. Lett.* **A13** (1998) 2601–2612, [arXiv:hep-th/9805067](#).
- [28] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, “The Hierarchy Problem and New Dimensions at a Millimeter,” *Phys. Lett.* **B429** (1998) 263–272, [arXiv:hep-ph/9803315](#).
- [29] L. Randall and R. Sundrum, “A Large Mass Hierarchy from a Small Extra Dimension,” *Phys. Rev. Lett.* **83** (1999) 3370–3373, [arXiv:hep-ph/9905221](#).
- [30] S. Weinberg, “Implications of Dynamical Symmetry Breaking: An Addendum,” *Phys. Rev.* **D19** (1979) 1277–1280.
- [31] L. Susskind, “Dynamics of Spontaneous Symmetry Breaking in the Weinberg- Salam Theory,” *Phys. Rev.* **D20** (1979) 2619–2625.
- [32] C. Bouchiat, J. Iliopoulos, and P. Meyer, “An Anomaly Free Version of Weinberg’s Model,” *Phys. Lett.* **B38** (1972) 519–523.
- [33] J. C. Pati and A. Salam, “Lepton Number as the Fourth Color,” *Phys. Rev.* **D10** (1974) 275–289.
- [34] H. Georgi and S. L. Glashow, “Unity of All Elementary Particle Forces,” *Phys. Rev. Lett.* **32** (1974) 438–441.
- [35] I.-G. Koh, J. Patera, and C. Rousseau, “Clebsch-Gordan Coefficients for $SU(5)$ Contains $SU(3) \times SU(2) \times U(1)$ Theories,” *J. Math. Phys.* **24** (1983) 1955.
- [36] M. A. Del Olmo, J. Patera, M. A. Rodriguez, and C. Rousseau, “Clebsch-Gordan Coefficients for $SU(5)$ Unification Models,” *J. Math. Phys.* **28** (1987) 258–271.

- [37] K. Inoue, A. Kakuto, and Y. Nakano, “Unification of the Lepton-Quark World by the Gauge Group $SU(6)$,” *Prog.Theor.Phys.* **58** (1977) 630.
- [38] A. Hartanto and L. T. Handoko, “Grand Unified Theory Based on the $SU(6)$ Symmetry,” *Phys. Rev.* **D71** (2005) 095013, [arXiv:hep-ph/0504280](#).
- [39] H. Fritzsch and P. Minkowski, “Unified Interactions of Leptons and Hadrons,” *Ann. Phys.* **93** (1975) 193–266.
- [40] I. G. Koh, J. Patera, and C. Rousseau, “Clebsch-Gordan Coefficients for E_6 and $SO(10)$ Unification Models,” *J. Math. Phys.* **25** (1984) 2863–2872.
- [41] P. Nath and R. M. Syed, “Complete Cubic and Quartic Couplings of 16 and $\overline{16}$ in $SO(10)$ Unification,” *Nucl. Phys.* **B618** (2001) 138–156, [arXiv:hep-th/0109116](#).
- [42] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, and N. Okada, “ $SO(10)$ Group Theory for the Unified Model Building,” *J. Math. Phys.* **46** (2005) 033505, [arXiv:hep-ph/0405300](#).
- [43] F. Gursey, P. Ramond, and P. Sikivie, “A Universal Gauge Theory Model Based on E_6 ,” *Phys. Lett.* **B60** (1976) 177.
- [44] G. W. Anderson and T. Blazek, “ E_6 Unification Model Building. I: Clebsch-Gordan Coefficients of $27 \otimes \overline{27}$,” *J. Math. Phys.* **41** (2000) 4808–4816, [arXiv:hep-ph/9912365](#).
- [45] G. W. Anderson and T. Blazek, “ E_6 Unification Model Building. II: Clebsch-Gordan Coefficients of $78 \otimes 78$,” *J. Math. Phys.* **41** (2000) 8170–8189, [arXiv:hep-ph/0006017](#).
- [46] G. W. Anderson and T. Blazek, “ E_6 Unification Model Building. III: Clebsch-Gordan Coefficients in E_6 Tensor Products of the 27 with Higher Dimensional Representations,” *J. Math. Phys.* **46** (2005) 013506, [arXiv:hep-ph/0101349](#).
- [47] M. Gell-Mann, P. Ramond, and R. Slansky, “Color Embeddings, Charge Assignments, and Proton Stability in Unified Gauge Theories,” *Rev.Mod.Phys.* **50** (1978) 721.
- [48] R. Slansky, “Group Theory for Unified Model Building,” *Phys. Rept.* **79** (1981) 1–128.
- [49] H. Georgi, “Lie Algebras in Particle Physics. From Isospin to Unified Theories,” *Front.Phys.* **54** (1982) 1–255.
- [50] J. Hisano, H. Murayama, and T. Yanagida, “Nucleon Decay in the Minimal Supersymmetric $SU(5)$ Grand Unification,” *Nucl. Phys.* **B402** (1993) 46–84, [arXiv:hep-ph/9207279](#).
- [51] **Super-Kamiokande** Collaboration, H. Nishino *et al.*, “Search for Nucleon Decay into Charged Anti-lepton plus Meson in Super-Kamiokande I and II,” *Phys.Rev.* **D85** (2012) 112001, [arXiv:1203.4030 \[hep-ex\]](#).
- [52] J. R. Ellis, D. V. Nanopoulos, and S. Rudaz, “GUTs 3: SUSY GUTs 2,” *Nucl. Phys.* **B202** (1982) 43.
- [53] N. Sakai and T. Yanagida, “Proton Decay in a Class of Supersymmetric Grand Unified Models,” *Nucl. Phys.* **B197** (1982) 533.
- [54] S. Weinberg, “Supersymmetry at Ordinary Energies. 1. Masses and Conservation Laws,” *Phys. Rev.* **D26** (1982) 287.
- [55] P. Nath, A. H. Chamseddine, and R. L. Arnowitt, “Nucleon Decay in Supergravity Unified Theories,” *Phys. Rev.* **D32** (1985) 2348–2358.

- [56] T. Goto and T. Nihei, “Effect of $RRRR$ Dimension Five Operator on the Proton Decay in the Minimal $SU(5)$ SUGRA GUT Model,” *Phys. Rev.* **D59** (1999) 115009, [arXiv:hep-ph/9808255](#).
- [57] J. Hisano, “Proton Decay in the Supersymmetric Grand Unified Models,” [arXiv:hep-ph/0004266](#).
- [58] M. Miura, “Search for Nucleon Decays in Super-Kamiokande,” *PoS ICHEP2010* (2010) 408.
- [59] A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, “Naturally Massless Higgs Doublets in Supersymmetric $SU(5)$,” *Phys. Lett.* **B115** (1982) 380.
- [60] B. Grinstein, “A Supersymmetric $SU(5)$ Gauge Theory with No Gauge Hierarchy Problem,” *Nucl. Phys.* **B206** (1982) 387.
- [61] K. Inoue, A. Kakuto, and H. Takano, “Higgs as (Pseudo)Goldstone Particles,” *Prog. Theor. Phys.* **75** (1986) 664.
- [62] E. Witten, “Symmetry Breaking Patterns in Superstring Models,” *Nucl. Phys.* **B258** (1985) 75.
- [63] T. Yanagida, “Naturally Light Higgs Doublets in the Supersymmetric Grand Unified Theories with Dynamical Symmetry Breaking,” *Phys. Lett.* **B344** (1995) 211–216, [arXiv:hep-ph/9409329](#).
- [64] Y. Kawamura, “Triplet-Doublet Splitting, Proton Stability and Extra Dimension,” *Prog. Theor. Phys.* **105** (2001) 999–1006, [arXiv:hep-ph/0012125](#).
- [65] N. Maekawa and T. Yamashita, “ E_6 Unification, Doublet-Triplet Splitting and Anomalous $U(1)_A$ Symmetry,” *Prog. Theor. Phys.* **107** (2002) 1201–1233, [arXiv:hep-ph/0202050](#).
- [66] M. Bando and T. Kugo, “Higgs Doublets as Pseudo Nambu-Goldstone Bosons in Supersymmetric $E(6)$ Unification,” *Prog. Theor. Phys.* **109** (2003) 87–101, [arXiv:hep-ph/0209088](#).
- [67] H. Georgi and C. Jarlskog, “A New Lepton - Quark Mass Relation in a Unified Theory,” *Phys. Lett.* **B86** (1979) 297–300.
- [68] H. Murayama, Y. Okada, and T. Yanagida, “The Georgi-Jarlskog Mass Relation in a Supersymmetric Grand Unified Model,” *Prog. Theor. Phys.* **88** (1992) 791–796.
- [69] S. P. Martin, “A Supersymmetry Primer,” [arXiv:hep-ph/9709356](#). in *Perspectives on Supersymmetry II*, ed. G. L. Kane (World Scientific, Singapore, 1997) pp. 1-153.
- [70] D. J. H. Chung *et al.*, “The Soft Supersymmetry-Breaking Lagrangian: Theory and Applications,” *Phys. Rept.* **407** (2005) 1–203, [arXiv:hep-ph/0312378](#).
- [71] H. Baer and X. Tata, *Weak Scale Supersymmetry -From Superfields to Scattering Events*. Cambridge University Press, 2006.
- [72] G. Ross and M. Serna, “Unification and Fermion Mass Structure,” *Phys. Lett.* **B664** (2008) 97–102, [arXiv:0704.1248 \[hep-ph\]](#).
- [73] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, and H. Okada, “Non-Abelian Discrete Symmetries in Particle Physics,” *Prog.Theor.Phys.Suppl.* **183** (2010) 1–163, [arXiv:1003.3552 \[hep-th\]](#).

- [74] S. F. King and G. G. Ross, “Fermion Masses and Mixing Angles from $SU(3)$ Family Symmetry,” *Phys. Lett.* **B520** (2001) 243–253, [arXiv:hep-ph/0108112](#).
- [75] N. Maekawa and T. Yamashita, “Horizontal Symmetry in Higgs Sector of GUT with $U(1)_A$ Symmetry,” *JHEP* **07** (2004) 009, [arXiv:hep-ph/0404020](#).
- [76] K. Inoue, “Generations of Quarks and Leptons from Noncompact Horizontal Symmetry,” *Prog. Theor. Phys.* **93** (1995) 403–416, [arXiv:hep-ph/9410220](#).
- [77] M. Gourdin, *Unitary Symmetries*. North-Holland Publishing Company - Amsterdam, 1967.
- [78] R. Gilmore, *Lie Groups, Lie Algebras, and some of their Applications*. Dover, New York, NY, 2005.
- [79] K. Inoue and N.-a. Yamashita, “Mass Hierarchy from $SU(1,1)$ Horizontal Symmetry,” *Prog. Theor. Phys.* **104** (2000) 677–689, [arXiv:hep-ph/0005178](#).
- [80] K. Inoue and N.-a. Yamashita, “Neutrino Masses and Mixing Matrix from $SU(1,1)$ Horizontal Symmetry,” *Prog. Theor. Phys.* **110** (2004) 1087–1094, [arXiv:hep-ph/0305297](#).
- [81] K. Inoue and N. Yamatsu, “Charged Lepton and Down-Type Quark Masses in $SU(1,1)$ Model and the Structure of Higgs Sector,” *Prog. Theor. Phys.* **119** (2008) 775–796, [arXiv:0712.2938 \[hep-ph\]](#).
- [82] K. Inoue and N. Yamatsu, “Strong CP Problem and the Natural Hierarchy of Yukawa Couplings,” *Prog. Theor. Phys.* **120** (2008) 1065–1091, [arXiv:0806.0213 \[hep-ph\]](#).
- [83] N. Yamatsu, “New Mixing Structures of Chiral Generations in a Model with Noncompact Horizontal Symmetry,” *Prog. Theor. Exp. Phys.* **2013** (2013) 023B03, [arXiv:1209.6318 \[hep-ph\]](#).
- [84] P. Fayet, “Spontaneously Broken Supersymmetric Theories of Weak, Electromagnetic and Strong Interactions,” *Phys. Lett.* **B69** (1977) 489.
- [85] M. Bento, L. J. Hall, and G. G. Ross, “Generalized Matter Parities from the Superstring,” *Nucl.Phys.* **B292** (1987) 400.
- [86] R. Barbier *et al.*, “ R -parity Violating Supersymmetry,” *Phys. Rept.* **420** (2005) 1–202, [arXiv:hep-ph/0406039](#).
- [87] K. Inoue, H. Kubo, and N. Yamatsu, “Vacuum Structures of Supersymmetric Noncompact Gauge Theory,” *Nucl. Phys.* **B833** (2010) 108–132, [arXiv:0909.4670 \[hep-th\]](#).
- [88] E. Cremmer and B. Julia, “The $SO(8)$ Supergravity,” *Nucl. Phys.* **B159** (1979) 141.
- [89] B. de Wit, P. G. Lauwers, R. Philippe, and A. Van Proeyen, “Noncompact $\mathcal{N} = 2$ Supergravity,” *Phys. Lett.* **B135** (1984) 295.
- [90] M. Magg and C. Wetterich, “Neutrino Mass Problem and Gauge Hierarchy,” *Phys. Lett.* **B94** (1980) 61.
- [91] G. Lazarides, Q. Shafi, and C. Wetterich, “Proton Lifetime and Fermion Masses in an $SO(10)$ Model,” *Nucl. Phys.* **B181** (1981) 287.
- [92] R. N. Mohapatra and G. Senjanovic, “Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation,” *Phys. Rev.* **D23** (1981) 165.

- [93] N. Cabibbo, “Unitary Symmetry and Leptonic Decays,” *Phys. Rev. Lett.* **10** (1963) 531–533.
- [94] M. Kobayashi and T. Maskawa, “CP Violation in the Renormalizable Theory of Weak Interaction,” *Prog. Theor. Phys.* **49** (1973) 652–657.
- [95] Z. Maki, M. Nakagawa, and S. Sakata, “Remarks on the Unified Model of Elementary Particles,” *Prog. Theor. Phys.* **28** (1962) 870–880.
- [96] K. Inoue, “Model of Generations of Quarks and Leptons,” *Prog. Theor. Phys. Suppl.* **123** (1996) 319–326.
- [97] M. Bando, T. Kuramoto, T. Maskawa, and S. Uehara, “Nonlinear Realization in Supersymmetric Theories,” *Prog. Theor. Phys.* **72** (1984) 313.
- [98] M. Bando, T. Kuramoto, T. Maskawa, and S. Uehara, “Nonlinear Realization in Supersymmetric Theories. 2,” *Prog. Theor. Phys.* **72** (1984) 1207.
- [99] M. Gell-Mann, P. Ramond, and R. Slansky, “Complex Spinors and Unified Theories,” *Print-80-0576* (CERN).
- [100] R. Foot, H. Lew, X. G. He, and G. C. Joshi, “Seesaw Neutrino Masses Induced by a Triplet of Leptons,” *Z. Phys.* **C44** (1989) 441.
- [101] R. N. Mohapatra *et al.*, “Theory of Neutrinos: A White Paper,” *Rept. Prog. Phys.* **70** (2007) 1757–1867, [arXiv:hep-ph/0510213](#).
- [102] **Particle Data Group** Collaboration, J. Beringer *et al.*, “Review of Particle Physics (RPP),” *Phys.Rev.* **D86** (2012) 010001.
- [103] J. Hisano, D. Kobayashi, T. Kuwahara, and N. Nagata, “Decoupling Can Revive Minimal Supersymmetric SU(5),” [arXiv:1304.3651 \[hep-ph\]](#).
- [104] E. Witten, “Global Aspects of Current Algebra,” *Nucl. Phys.* **B223** (1983) 422–432.